Midterm

University of Toronto

CSC418H
Computer Graphics

Fall 2013

Instructor: Luke Moore

Wednesday, October 23, 2013
Duration: 60 minutes

This is a closed book exam (no calculators, PDAs, laptops or other examination aids)

Note: It is a violation of the University’s Code of Behaviour on Academic Matters to cheat on an examination. Cheating includes copying another student’s work, possessing unauthorized examination aids, having someone else take your examination, or knowingly allowing your work to be copied.
Question 1: Consider the 3D implicit hyperboloid surface defined by the following equation:

\[ f_1(x, y, z) = \frac{x^2}{2} + \frac{y^2}{4} - \frac{z^2}{8} - 1 = 0 \]

a) [1 marks] Show that the point (2, 2, 4) lies on the surface.

b) [3 marks] Compute the unit surface normal at the point in (a).

c) [4 marks] Show that the points defined by this parametric equation lie on the surface:

\[ f_2(u, v) = (\sqrt{2}\cos v \sqrt{u^2 + 1}, 2\sin v \sqrt{u^2 + 1}, \sqrt{8}u) \]

where \( u \in [0, \infty], v \in [0, 2\pi] \)
Question 2: [10 marks] Find the 3x3 homogeneous matrix that transforms the 2D vertices of the object on the left to the object on the right in the figure below. Express the matrix as a composition of elementary transformations.
Question 3: [8 marks] A point is drawn at the origin. It is viewed through a camera that has the following world-to-camera matrix:

\[
\begin{bmatrix}
2 & -1 & 0 & 3 \\
1 & 2 & 0 & -1 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The object containing the point is transformed with the following transformation matrix:

\[
\begin{bmatrix}
0 & -1 & 0 & 5 \\
1 & 0 & 0 & 3 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

This simple projective transformation is used:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\]

Where does the point appear in screen coordinates (give the x,y position)? Show your intermediate work.
Question 4: Coordinate Transformations
   a) [8 marks] Consider a two-link, two-dimensional robotic arm with the following properties:
      • Its base is affixed to the origin of the world coordinate system;
      • Its first link has length $l_1$ and can rotate freely about the origin by angle $\theta_1$;
      • Its second link has length $l_2$ and can rotate freely by an angle $\theta_2$ about the links’ common joint.

Suppose we attach a local coordinate system to the end of the second joint, as shown in the figure above, with its $b$-axis aligned with the joint and its origin at the joint’s end.

Express the transformation $T_1$ that maps local coordinates $(a, b)$ to 2D world coordinates $(u, v)$ as a product of elementary homogeneous transformation matrices. You should show the elements of each matrix and indicate the transformations the matrix corresponds to. You do not need to perform any matrix multiplications.

$T_1 =$
b) [8 marks] Now suppose that the planar arm in (a) actually lives in a 3D world and has an extra degree of freedom: in addition to rotating its links by angles $\theta_1$ and $\theta_2$, the arm’s $uv$-plane can rotate about the $y$-axis of the world coordinate system by an angle $\theta_3$:

![Diagram of a 3D arm with additional plane rotation]

As shown in the figure, the arm’s base is at the origin of the 3D world coordinate system and the $v$-axis of the arm’s plane is aligned with the $y$-axis.

Express the transformation $T_2$ that maps local coordinates $(a, b)$ to 3D world coordinates $(x, y, z)$ as a product of elementary homogeneous transformation matrices.

$T_2 =$
Question 5:
   a) [2 marks] In words, what is a backface?

   b) [4 marks] Explain mathematically how to perform backface culling for a triangle with vertices $\overrightarrow{p_1}, \overrightarrow{p_2}$, and $\overrightarrow{p_3}$ and outward-facing normal $\overrightarrow{n}$, where the eye of the camera is at location $\overrightarrow{e}$ with a viewing direction of $\overrightarrow{g}$?

   c) [2 marks] Sketch a simple diagram to show one example in which a triangle is not visible at all but would not be removed by backface culling?