CSC236 tutorial exercises

(Best before 11 am, Monday October 15th)

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Here are your tutorial sections:

Surname	Section	Room	TA
A–F	Day 1 (11:00 am)	LM162	Yuval
G–Li	Day 2 (11:00 am)	BA2139	Lila
Lo–Si	Day 3 (11:00 am)	BA2145	Oles
So–Z	Day 4 (11:00 am)	BA2155	Lalla
A–H	Evening 1 (8:00 pm)	BA1190	Colin
I–M	Evening 2 (8:00 pm)	BA2135	Norman
N–Z	Evening 3 (8:00 pm)	BA2139	Feyyaz

These exercises are intended to give you practice with unwinding and proving recurrences.

1. Consider the recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 1 + T(\lceil n/2 \rceil) & \text{if } n > 1 \end{cases}$$

   Use complete induction to prove that for every positive natural number n, $T(n) \geq c\lg(n)$, for some positive real constant c.

2. Unwind the recurrence from the previous question in the case where $n = 2^k$ for some positive integer k (see annotated slides from October 11th or 12th). Use mathematical induction on k to prove that $T(2^k) = k + 1$.

3. Consider another recurrence:

$$G(n) = \begin{cases} 1 & \text{if } n < 2 \\ 1 + G(n-1) + G(n-2) & \text{if } n \geq 2 \end{cases}$$

   Unwind the recurrence **carefully**, following the pattern below, for some n that is comfortably greater than 1:

$$\begin{aligned} G(n) &= 1 + G(n-1) + G(n-2) \\ &= 1 + (1 + G(n-2) + G(n-3)) + G(n-2) = 2 + 2G(n-2) + G(n-3) \\ &= 2 + 2(1 + G(n-3) + G(n-4)) + G(n-3) = 4 + 3G(n-3) + 2G(n-4) \\ &= 4 + 3(1 + G(n-4) + G(n-5)) + 2G(n-4) = 7 + 5G(n-4) + 3G(n-5) \\ &\vdots \end{aligned}$$

   Can you see a pattern that leads to a guess at a closed form for $G(n)$?