CSC236 fall 2012 recursion, induction, correctness

Danny Heap heap@cs.toronto.edu BA4270 (behind elevators) http://www.cdf.toronto.edu/~heap/236/F12/ 416-978-5899

Using Introduction to the Theory of Computation, Chapter 2

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Outline

binary search

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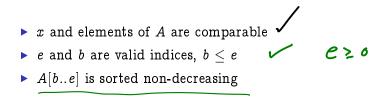
recursive binary search

```
def recBinSearch(x, A, b, e) :
     if b == e:
۱.
       if x \le A[b] :
2.
3.
        return b
4.
      else :
5,
         return e + 1
6.
   else :
7.
      m = (b + e) // 2 \# midpoint
 g. if x \le A[m] :
 q.
         return recBinSearch(x, A, b, m)
     else :
 10.
      return recBinSearch(x, A, m+1, e)
  11+
```

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conditions, pre- and post-



 $\operatorname{RecBinSearch}(x, A, b, e)$ terminates and returns index p

$$lacksim b \leq p \leq e+1$$
 (4)

$$\blacktriangleright \ b$$

$$\blacktriangleright \ p \leq e \Rightarrow x \leq A[p] \quad \checkmark$$

(except for boundaries, returns p so that $A[p-1] < x \leq A[p]$)

Computer Science UNIVERSITY OF TORON precondition \Rightarrow termination and postcondition Proof: induction on n = e - b + 1

Base case, n = 1: Terminates because there are no loops or further calls, returns $x \leq A[b = p] \Leftrightarrow p \leq b = e$ is returned. $x > A[b = p - 1] \Leftrightarrow p = b + 1$ returned, so postcondition satisfied. Notice that the choice forces if-and-only-if.

Induction step: Assume n > 1 and that the postcondition is satisfied for inputs of size $1 \le k < n$ that satisfy the precondition. Call RecBinSearch(A,x,b,e) when $m = \underbrace{e+b}_{2}$ n = e - b + 1 > 1. Since b < e in this case, the test on line 1 fails, and line 7 executes. Exercise: $b \le m < e$ in this case. There are two cases, according to whether $x \le A[m]$ or x > A[m].

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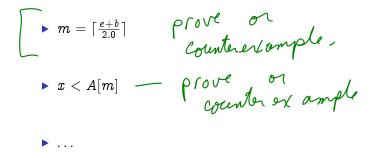
Case 1:
$$x \leq A[m] \ll$$

 $O \leq m-b+1 \leq e-b+1 \quad So$
 $w_y ex$
Show that [H applies to RBS(x,A,b,m)] K
Translate the postcondition to RBS(x,A,b,m), ||| (allo
 $b \leq \rho \leq m+1$
 $b \leq \rho \leq m+1$
 $b \leq \rho = A[\rho-1] \leq X$ (Soly Induct.
 $\rho \leq m \Rightarrow \chi \leq A[\rho]$ (Soly Induct.
 $P \leq m \Rightarrow \chi \leq A[\rho]$ (Soly Induct.
 $Hypoth$
Show that RBS(x,A,b,e) satisfies postcondition
 $By^{[H]} \Rightarrow b \leq \rho \leq m+1 \leq e \neq e+1$
 $b \leq \rho \Rightarrow A[\rho-1] \leq \chi \# by IH$
 $b \leq \rho \Rightarrow A[\rho-1] \leq \chi \# by IH$
 $p \leq e \Rightarrow \chi \leq A[\rho] must show \chi \leq A[\rho]$
 $\psi = m+1 \Rightarrow then \chi \leq A[m] = by A[\rho] = hyft
 $\psi = m, \forall len \chi \leq A[\rho] \leq hyft$$

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Case 2: x > A|m|Nacis. Show that IH applies to $\mathbb{RBS}(x,A,m+1,e)$ ▶ Translate postcondition to RBS(x,A,m+1,e) 1 H mean oh < m+l ≤ p ≤ e+1 V · p = e = x = ACP] · m+1 ACP-J < X a ► Show that RBS(x,A,b,e)• $b \leq \rho \leq e+1$ → $b \neq b \neq 1H$ and $m \geq b$ · p≤e' ⇒ x ≤ A[P] * by 1H. · Show b <P => ACP-IKX • p > m+1, then $A \ge p-1 \le \chi$ by H• $p \le m+1 \Longrightarrow$ p = m+1, then $\chi > A \ge m = A \ge 1$

what could go wrong?



Either prove correct, or find a counter-example

