SLOGs you'll

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time complexity

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Using Introduction to the Theory of Computation, Chapter 3

Outline

complexity of recursive functions

Notes

binary search

Recursive $T(n)$

T — monotonic? $T(x) \geq T(y)$, $x \geq y$

$T(n)$

```
def recBinSearch(x, A, b, e) :
  if b == e :
    if x <= A[b] :
      return b
    else :
      return e + 1
  else :
    m = (b + e) // 2 # midpoint
    if x <= A[m] :
      return recBinSearch(x, A, b, m)
    else :
      return recBinSearch(x, A, m+1, e)
```

c_1, c_2, c_3, c_4 — max c' — 1

c_5, c_6 — 1

$T(m-b+1)$

$T(e-(m+1)+1) = T(e-m)$.

$n = \underbrace{e-b+1}$

$$T(n) = \begin{cases} 1 & n=1 \\ 1 + \max\{T(m-b+1), T(e-m)\} \end{cases}$$

$\lceil \frac{n}{2} \rceil$ $\lfloor n/2 \rfloor$

Lower bound on $T(n)$… by unwinding

Suppose $n = 2^k$ some $k \in \mathbb{N}$.

$\log_2 n = \lg n$

Then $T(n) = 1 + T(\frac{n}{2}) = 1 + T(2^{k-1})$

$= 1 + 1 + T(2^{k-2})$

$\vdots$

$= k + T(2^{k-k}) = k+1 = \lg n + 1$

$T(n) \in O(\lg)$
$\in \Omega(\lg)$

big-oh
show $B \in \mathbb{N}$, $c \in \mathbb{R}^+$
s.t. $\forall n \geq B$, $T(n) \leq c \lg(n)$?

$T(n) \in \Theta(\lg)$ means ~~$T \in O \cap \Omega(n)$~~ $T \in O(\lg) \cap \Omega(\lg)$

find $B \in \mathbb{N}$, $c \in \mathbb{R}^+$, so that $\forall n \geq B$, $T(n) \geq c \lg(n)$.

Lower bound on $T(n)$...by proof

Want to show $T(n) \geq c \lg(n)$

Case $n = 1$, $T(n) = 1 \geq c \lg(1) = 0$

assume $n \in \mathbb{N}^+$ and that $T(i) \geq c \lg(i)$ $\forall 1 \leq i < n$. Must show that $T(n) \geq c \lg(n)$.

during proof, we figure out B, C

$\lg(n/2)$

$\lg n - \underline{\lg 2}$

Upper bound on $T(n)$

trouble!

Notes

$T(\underbrace{m-b+1}_{\lceil n/2 \rceil})$, $T(\underbrace{e-m}_{})$ vs $T(n)$

$T(e-b+1)$

$$\left\lfloor \frac{e+b}{2} \right\rfloor - b + 1$$

$$= \left\lfloor \frac{e+b}{2} - b + 1 \right\rfloor$$

\# $\lfloor x + z \rfloor = \lfloor x \rfloor + z$

\# $z \in \mathbb{Z} \rightarrow$ deserve MP

\#

$$= \left\lfloor \frac{e-b+2}{2} \right\rfloor = \left\lfloor \frac{e-b+1+1}{2} \right\rfloor$$

$$= \left\lceil \frac{e-b+1}{2} \right\rceil$$

\# $\left\lfloor \frac{k+1}{2} \right\rfloor = \left\lceil \frac{k}{2} \right\rceil$

\# $\forall k \in \mathbb{N}$

$$= \lceil n/2 \rceil$$

Notes

$T(e-m)$ vs $T(e-b+1)$.

$$e - \left\lfloor \frac{e+b}{2} \right\rfloor = e + \left\lceil -\frac{e+b}{2} \right\rceil \quad \# \; -\lfloor x \rfloor = \lceil -x \rceil$$

$$= \left\lceil e - \frac{e+b}{2} \right\rceil \quad \# \; \lceil k + x \rceil = k + \lceil x \rceil$$

$$\# \; \forall k \in \mathbb{N}$$

$$= \left\lceil \frac{e-b}{2} \right\rceil = \left\lfloor \frac{e-b+1}{2} \right\rfloor \quad \# \; \left\lfloor \frac{k+1}{2} \right\rfloor = \left\lceil \frac{k}{2} \right\rceil$$

$$= \lfloor n/2 \rfloor$$

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + \max\{T(\lfloor n/2 \rfloor), T(\lceil n/2 \rceil)\} \end{cases}$$

assume now, prove later, that max above is $T(\lceil n/2 \rceil)$.