

SLOGs → you'll

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time complexity

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Using Introduction to the Theory of Computation,  
Chapter 3

# Outline

complexity of recursive functions

Notes

# binary search

Recursive  $T(n)$

$T$  - monotonic?  $T(x) \geq T(y)$   
 $x \geq y$

$T(n)$

```
def recBinSearch(x, A, b, e) :
```

```
    if b == e :  $-c_1$ 
```

```
        if x <= A[b] :  $-c_2$ 
```

```
            return b  $-c_3$ 
```

```
        else :
```

```
            return e + 1  $c_4$ 
```

```
    else :
```

```
        [m = (b + e) // 2 # midpoint  $-c_5$ ] 1
```

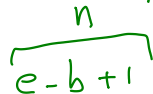
```
        if x <= A[m] :  $c_6$ 
```

```
            return recBinSearch(x, A, b, m)  $-T(m-b+1)$ 
```

```
        else :
```

```
            return recBinSearch(x, A, m+1, e)  $-T(e-(m+1)+1)$   
 $= T(e-m)$ 
```

max  $c_i$



$$T(n) = \begin{cases} 1 & n=1 \\ \max\{T(n-b+1), T(e-m)\} & \text{otherwise} \end{cases}$$

$$\lceil \frac{n}{2} \rceil \quad \lfloor \frac{n}{2} \rfloor$$

# Lower bound on $T(n)$ ... by unwinding

$$\text{Then } \underline{T(n)} = \underline{1 + T(\frac{n}{2})} = \underline{1 + T(2^{k-1})}$$

$$= 1 + 1 + T(2^{k-2})$$

⋮

$$= \underline{k+1} = \underline{\lg n + 1}$$

Suppose  $n = 2^k$   
some  $k \in \mathbb{N}$ .

$$\underline{\log_2 n \equiv \lg n}$$

~~$T(n) \in O(\lg n)$~~   
 $\in \Omega(\lg n)$

big-oh snow  $B, \in \mathbb{N}, c \in \mathbb{R}^+$

s.t.  $\forall n \geq B, T(n) \leq c \lg(n)$   
find  $B \in \mathbb{N}$

$T(n) \in \Theta(\lg)$  means

~~$T \in O(\lg) \wedge \Omega(\lg)$~~   
 $T \in \underline{O(\lg)} \wedge \underline{\Omega(\lg)}$

$c \in \mathbb{R}^+$ , so that  
 $\forall n \geq B, T(n) \geq c \lg(n)$

## Lower bound on $T(n)$ ... by proof

Want to show  $T(n) \geq c \lg(n)$

Case  $n=1$ ,  $T(n)=1 \geq c \lg(1)=0$

Assume  $n \in \mathbb{N}^+$  and that  $T(i) \geq c \lg(i)$   
 $\forall 1 \leq i < n$ . Must show  
that  $T(n) \geq c \lg(n)$ .

during  
proof,  
we figure  
out B, C

$\lg(n/2)$   
 $\lg n - \lg 2$



# Upper bound on $T(n)$

trouble!

Notes

$$T\left(\frac{m-b+1}{\lceil n/2 \rceil}\right), \quad T(e-m) \quad \text{vs} \quad T(n)$$

$$\downarrow \left\lfloor \frac{e+b}{2} \right\rfloor - b + 1$$

$$= \left\lfloor \frac{e+b}{2} - b + 1 \right\rfloor$$

$$= \left\lfloor \frac{e-b+2}{2} \right\rfloor = \left\lfloor \frac{e-b+1+1}{2} \right\rfloor$$

$$= \left\lfloor \frac{e-b+1}{2} \right\rfloor$$

#  $\lfloor x+z \rfloor = \lfloor x \rfloor + z$   
#  $z \in \mathbb{Z} \rightarrow$  leave MP  
#  $\forall k \in \mathbb{N} \quad \left\lfloor \frac{k+1}{2} \right\rfloor = \left\lfloor \frac{k}{2} \right\rfloor$

$$= \left\lceil \frac{n}{2} \right\rceil$$



## Notes

$T(e-m)$  vs  $T(e-b+1)$ .

$$\begin{aligned}
 e - \lfloor \frac{e+b}{2} \rfloor &= e + \lceil \frac{-e-b}{2} \rceil \neq -\lfloor x \rfloor = \lceil -x \rceil \\
 &= \lceil e - \frac{e+b}{2} \rceil \neq \lceil k+x \rceil = k + \lceil x \rceil \\
 &= \lceil \frac{e-b}{2} \rceil = \lfloor \frac{e-b+1}{2} \rfloor \neq \lfloor \frac{k+1}{2} \rfloor = \lfloor \frac{k}{2} \rfloor
 \end{aligned}$$

$\neq \forall k \in \mathbb{N}$

$$T(n) = \begin{cases} 1 & n = 1 \\ t + \max \{ T(\lfloor n/2 \rfloor), T(\lceil n/2 \rceil) \} & = \lfloor n/2 \rfloor \end{cases}$$

assume now, prove later,  
that max above is  $T(\lceil n/2 \rceil)$ .

