

term test #1 ✓
assignment 1 ✓
SLOG?

CSC236 fall 2012

time complexity

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BA4270 (behind elevators)

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Using Introduction to the Theory of Computation,
Chapter 3

Outline

complexity of recursive functions

Notes

binary search

Recursive $T(n)$

* must prove T is increasing
 $n = e - b + 1$

$T(n)$

```
def recBinSearch(x, A, b, e) :
```

```
    if b == e :  $c_1$ 
```

```
        if x <= A[b] :  $c_2$ 
```

```
            return b
```

```
        else :
```

```
            return e + 1  $c_4$ 
```

```
    else :
```

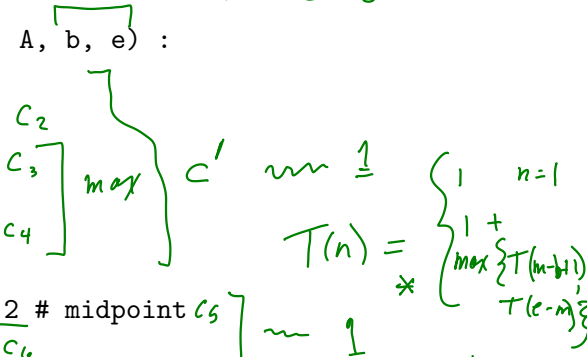
```
        m = (b + e) // 2 # midpoint  $c_5$ 
```

```
        if x <= A[m] :  $c_6$ 
```

```
            return recBinSearch(x, A, b, m) -  $T(m - b + 1)$ 
```

```
        else :
```

```
            return recBinSearch(x, A, m+1, e) -  $T(e - (m+1) + 1)$   
                =  $T(e - m)$ 
```



Lower bound on $T(n)$... by unwinding $= 1 + T(n/2)$

Suppose $n = 2^{2^k}$, some $k > 0$

$$\begin{aligned} T(n) &= 1 + T(2^{k-1}) \\ &= 2 + T(2^{k-2}) \\ &= 3 + T(2^{k-3}) \end{aligned}$$

"after" some $B \in \mathbb{N}$ $\Theta(\lg)$: # probably an inductive proof
some $T(n) \leq c \lg n$: # need.

$$\begin{aligned} \text{and } T(n) &\geq c' \lg n = k + T(2^{k-k}) & 1 + T(n/2) \\ &= k + 1 = \lg n + 1 & = \underbrace{1 + 1 + T(n/4)} \end{aligned}$$

guess that $\forall n \in \mathbb{N}, T(n) \stackrel{N}{\approx} c \lg n$

$$\begin{aligned} 1 + T(2^{k-1}) &= 1 + (1 + T(2^{k-2})) \\ &= 2 + (1 + T(2^{k-3})) \end{aligned}$$

Lower bound on $T(n)$... by proof want to show
 $T(n) \geq c \lg n$

Scratch (induction).

assume $n \in \mathbb{N}^+$ and $T(i) \geq c \lg i \quad \forall i, 1 \leq i < n$

$$\text{So } T(n) = 1 + T(\lceil n/2 \rceil) \quad \begin{array}{l} \# \text{ assuming} \\ \# T \text{ is } \uparrow. \end{array}$$

$$\geq 1 + c \lg \lceil n/2 \rceil \quad \# 1 \leq \lceil n/2 \rceil < n^*$$

$$\geq 1 + c \lg \left(\frac{n}{2}\right) \quad \# \lg \text{ is increasing}$$

$$= 1 + c(\lg n - \lg 2)$$

$$= 1 + c(\lg n - 1)$$

$$= 1 - c + c \lg n$$

$$\geq c \lg n$$

$\# 1 \geq c.$



Upper bound on $T(n)$

trouble!



Notes $n = e - b + 1$ versus $m - b + 1$ and $e - m$

$$m - b + 1 = \left\lceil \frac{e + b}{2} \right\rceil - b + 1$$

$$= \left\lceil \frac{e + b}{2} - b + 1 \right\rceil$$

$$= \left\lceil \frac{e - b + 2}{2} \right\rceil$$

$$= \left\lceil \frac{e - b + 1 + 1}{2} \right\rceil = \left\lceil \frac{e - b + 1}{2} \right\rceil$$

$$= \left\lceil \frac{n}{2} \right\rceil$$

$$\# \left\lceil \lfloor x \rfloor + k \right\rceil = \left\lceil \lfloor x + k \rfloor \right\rceil$$

$$\# \left\lceil \frac{k+1}{2} \right\rceil = \left\lceil \frac{k}{2} \right\rceil$$

$$\# k \in \mathbb{N}$$



Notes

$$\begin{aligned}
 e - m &= e - \left\lfloor \frac{e+b}{2} \right\rfloor \\
 &= e + \left\lceil -\frac{e+b}{2} \right\rceil \quad \# \quad -\lfloor x \rfloor = \lceil x \rceil \\
 &= \left\lceil e - \frac{e+b}{2} \right\rceil = \left\lceil \frac{e-b}{2} \right\rceil = \left\lfloor \frac{e-b+1}{2} \right\rfloor \\
 &= \left\lfloor \frac{n}{2} \right\rfloor \quad \# \quad \left\lceil \frac{k}{2} \right\rceil = \left\lfloor \frac{k+1}{2} \right\rfloor \\
 &\quad \# \quad k \in \mathbb{N}.
 \end{aligned}$$

$$T(n) = \begin{cases} 1 & n=1 \\ 1 + \sum_{\substack{e \\ e}} \max \left\{ \underline{T(\lceil n/2 \rceil)}, T(\lfloor n/2 \rfloor) \right\} \end{cases}$$

must T is increasing

