term test #1 ✓
assignment 1 ✓
SLOG?

# CSC236 fall 2012

## time complexity

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Using Introduction to the Theory of Computation, Chapter 3

# Outline

complexity of recursive functions

Notes

# binary search

Recursive $T(n)$

* must prove $T$ is increasing

$T(n)$

$n = e - b + 1$

```
def recBinSearch(x, A, b, e) :
  if b == e :
    if x <= A[b] :
      return b
    else :
      return e + 1
  else :
    m = (b + e) // 2 # midpoint
    if x <= A[m] :
      return recBinSearch(x, A, b, m)
    else :
      return recBinSearch(x, A, m+1, e)
```

$c_1$, $c_2$, $c_3$, $c_4$ — max — $c'$ ~ 1

$c_5$, $c_6$ ~ 1

$T(m-b+1)$

$T(e-(m+1)+1) = T(e-m)$

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + \max\{T(m-b+1),\ T(e-m)\} & \end{cases}$$

# Lower bound on $T(n)$... by unwinding

$= 1 + T(n/2)$

Suppose $\underline{n = 2^k}$, some $k > 0$

$T(n) = 1 + T(2^{k-1})$

$= 2 + T(2^{k-2})$

$= 3 + T(2^{k-3})$

$\vdots$ # probably an inductive proof

$\vdots$ # need.

"after" some $B \in \mathbb{N}$ $T(n) \le c \lg n$ and $T(n) \ge c' \lg n$ $\Theta(\lg)$

$= k + T(2^{k-k})$

$1 + T(n/2)$

$= \underbrace{1 + 1} + T(n/4)$

$= k + 1 = \lg n + 1$

guess that $\forall n \in \mathbb{N}$, $T(n) \overset{\sim}{=} c \lg n$

$1 + T(2^{k-1}) = 1 + (1 + T(2^{k-2}))$

$= 2 + (1 + T(2^{k-3}))$

# Lower bound on $T(n)$... by proof

want to show $T(n) \geq c \lg n$

Scratch (induction).

assume $n \in \mathbb{N}^+$ and $T(i) \geq c \lg i \quad \forall i, 1 \leq i < n$

So $T(n) = 1 + T(\lceil n/2 \rceil)$ # assuming # $T$ is $\uparrow$.

$\geq 1 + c \lg \lceil n/2 \rceil$ # $1 \leq \lceil n/2 \rceil < n$

$\geq 1 + c \lg \left(\frac{n}{2}\right)$ # $\lg$ is increasing

$= 1 + c(\lg n - \lg 2)$

$= 1 + c(\lg n - 1)$

$= 1 - c + c \lg n$

$\geq c \lg n$ # $1 \geq c$.

# Upper bound on $T(n)$

trouble!

# Notes

$n = e - b + 1$ versus $m - b + 1$ and $e - m$

$$m - b + 1 = \left\lfloor \frac{e+b}{2} \right\rfloor - b + 1$$

$$= \left\lfloor \frac{e+b}{2} - b + 1 \right\rfloor$$

# $\lfloor x \rfloor + k \in \mathbb{Z}$ $= \lfloor x + k \rfloor$

$$= \left\lfloor \frac{e-b+2}{2} \right\rfloor$$

$$= \left\lfloor \frac{e-b+1+1}{2} \right\rfloor = \left\lceil \frac{e-b+1}{2} \right\rceil$$

# $\left\lfloor \frac{k+1}{2} \right\rfloor = \left\lceil \frac{k}{2} \right\rceil$

# $k \in \mathbb{N}$

$$= \left\lceil \frac{n}{2} \right\rceil$$

## Notes

$$e - m = e - \left\lfloor \frac{e+b}{2} \right\rfloor$$

$$= e + \left\lceil -\frac{e+b}{2} \right\rceil \qquad \# \; -\lfloor x \rfloor = \lceil -x \rceil$$

$$= \left\lceil e - \frac{e+b}{2} \right\rceil = \left\lceil \frac{e-b}{2} \right\rceil = \left\lfloor \frac{e-b+1}{2} \right\rfloor$$

$$\# \left\lceil \frac{k}{2} \right\rceil = \left\lfloor \frac{k+1}{2} \right\rfloor$$

$$\# \; k \in \mathbb{N}.$$

$$= \lfloor n/2 \rfloor$$

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + \max\{\underline{T(\lceil n/2 \rceil)}, T(\lfloor n/2 \rfloor)\} & \end{cases}$$

must $T$ is increasing