CSC236 fall 2012

subtle induction

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Using Introduction to the Theory of Computation, Section 1.2–1.3

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Outline

Well-ordering

Higher, and more, base cases

Well-ordering example

$\forall n, m \in \mathbb{N}, n \neq 0,\ R = \{r \in \mathbb{N} \mid \exists q \in \mathbb{N}, m = qn + r\}$ has a smallest element

This is the main part of proving the existence of a unique quotient and remainder:

$$\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \wedge 0 \leq r < n$$

The course notes use Mathematical Induction. Well-ordering is shorter and clearer.

Principle of well-ordering

Every non-empty subset of $\mathbb{N}$ has a smallest element

Is there something similar for $\mathbb{Q}$ or $\mathbb{R}$?

For a given pair of natural numbers $m, n \neq 0$ does the set R satisfy the conditions for well-ordering?

$$R = \{r \in \mathbb{N} \mid \exists q \in \mathbb{N}, m = qn + r\}$$

If so, we still need to be sure that

1. $0 \leq r < n$
2. That q and r are unique — no other natural numbers would work

...in order to have

$$\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \wedge 0 \leq r < n$$

Every non-empty subset of $\mathbb{N}$ has a smallest element

$\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \wedge 0 \leq r < n$

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Every non-empty subset of $\mathbb{N}$ has a smallest element

$P(n)$: Every round-robin tournament with n players that has a cycle has a 3-cycle

Claim: $\forall n \in \mathbb{N} - \{0, 1, 2\}, P(n)$.

If there is a cycle $p_1 > p_2 > p_3 \ldots > p_n > p_1$, can you find a shorter one?

Every non-empty subset of $\mathbb{N}$ has a smallest element

$P(n)$: Every round-robin tournament with n players that has a cycle has a 3-cycle

Claim: $\forall n \in \mathbb{N} - \{0, 1, 2\}, P(n)$.

Every non-empty subset of $\mathbb{N}$ has a smallest element

$P(n)$: Every round-robin tournament with n players that has a cycle has a 3-cycle

$2^n > 10n$

Where do we start?

It's not true for several low values of n. You could re-write the predicate as $P'(n) : 2^{n+6} > 10(n+6)$, but why not just start later?

$2^n > 10n$

Where do we start?

$3^n \geq n^3$

Check your induction step

Look at the graph.

$3^n \geq n^3$

Check your induction step

Notes