#### CSC236 fall 2012

subtle induction

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Using Introduction to the Theory of Computation, Section 1.2-1.3

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## Outline

Well-ordering

Higher, and more, base cases



Well-ordering example  $\forall n, m \in \mathbb{N}, n \neq 0, R = \{r \in \mathbb{N} \mid \exists q \in \mathbb{N}, m = qn + r\}$  has a smallest element

This is the main part of proving the existence of a unique quotient and remainder:

 $\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \land 0 \leq r < n$ 

The course notes use Mathematical Induction. Well-ordering is shorter and clearer.



## Principle of well-ordering

Every non-empty subset of  $\mathbb{N}$  has a smallest element

Is there something similar for  $\mathbb{Q}$  or  $\mathbb{R}$ ?

For a given pair of natural numbers  $m, n \neq 0$  does the set R satisfy the conditions for well-ordering?

 $R=\{r\in\mathbb{N}\mid \exists\,q\in\mathbb{N},m=qn+r\}$ 

If so, we still need to be sure that

- 1.  $0 \le r < n$
- That q and r are unique no other natural numbers would work
- ... in order to have

 $orall m \in \mathbb{N}, orall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \land 0 \leq r < n$ 

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Every non-empty subset of N has a smallest element  $\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \land 0 \leq r < n$ 



Every non-empty subset of N has a smallest element  $\forall m \in \mathbb{N}, \forall n \in \mathbb{N} - \{0\}, \exists q, r \in \mathbb{N}, m = qn + r \land 0 \leq r < n$ 



Every non-empty subset of  $\mathbb{N}$  has a smallest element P(n): Every round-robin tournament with n players that has a cycle has a 3-cycle

Claim:  $\forall n \in \mathbb{N} - \{0, 1, 2\}, P(n).$ 

If there is a cycle  $p_1 > p_2 > p_3 \ldots > p_n > p_1$ , can you find a shorter one?



Every non-empty subset of  $\mathbb{N}$  has a smallest element P(n): Every round-robin tournament with n players that has a cycle has a 3-cycle

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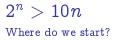
# Every non-empty subset of $\mathbb{N}$ has a smallest element P(n): Every round-robin tournament with n players that has a cycle has a 3-cycle



### $2^n > 10n$ Where do we start?

It's not true for several low values of n. You could re-write the predicate as  $P'(n): 2^{n+6} > 10(n+6)$ , but why not just start later?







 $3^n \geq n^3$ Check your induction step

Look at the graph.



 $3^n \ge n^3$ 

Check your induction step



## Notes

