CSC236 fall 2012

complete induction

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Using Introduction to the Theory of Computation, Section 1.3

Outline

Principle of complete induction

Examples of complete induction

Complete Induction

another flavour needed

Every natural number greater than 1 has a prime factorization

Try some examples

How does the factorization of 8 help with the factorization of 9?

More dominoes

DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO DOMINO

$$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$$

If all the previous cases always implies the current case
then all cases are true

Every natural number greater than 1 has a prime factorization

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Trees

definitions, page 32

- A tree is a directed graph
- A non-empty tree has a root node, r, such that there is exactly one path from r to any other node.
- If a tree has an edge (u, v), then u is v's parent, v is u's child.
- Two nodes with the same parent are called siblings.
- A node with no children is called a leaf. A non-leaf is called an internal node.
- Binary trees have nodes with ≤ 2 children, and children are labelled either left or right.
- Internal nodes of full binary trees have 2 children.

Tree examples

know your trees. . .

Every full binary tree, except the zero tree, has an odd number of nodes

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After a certain natural number n, every postage can be made up by combining $3-$ and $5-$ cent stamps