Tutorials announcement

we have rooms for Mondays only, so tutorials begin ... next Monday

lectures → WF

CSC236 fall 2012

complete induction

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Using Introduction to the Theory of Computation, Section 1.3

Outline

Principle of complete induction

Examples of complete induction

Complete Induction

another flavour needed

Every natural number greater than 1 has a prime factorization

$2 = 2$ $\quad 4 = 2 \times 2$

$3 = 3$ $\quad 5 = 5$ $\quad 6 = 2 \times 3$ $\quad 7 = 7$

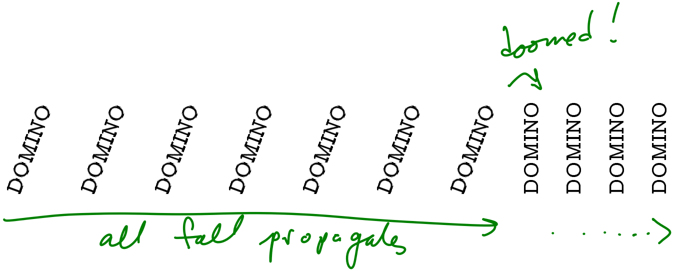
$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

Try some examples

How does the factorization of 8 help with the factorization of 9?

More dominoes



$$(\forall n \in \mathbb{N}, \{P(0), \ldots, P(n-1)\} \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$$

If all the previous cases always implies the current case
then all cases are true

$P(n)$: ~~Every natural number greater than 1~~ n has a prime factorization

$\mathbb{N} - \{0, 1\}$

$(\forall n \in \mathbb{N}, \{P(2), \ldots, P(n-1)\} \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Assume that $n \in \mathbb{N} - \{0,1\}$ and that for all natural numbers $1 < j < n$, $P(j)$ is true (j has prime factorization).

Then n is either prime or not prime.

Case 1: n is prime. Then $n = n$ is its own prime factorization!

Case 2: n is not prime. Then n has at least 3 factors, 1, n, and $x \neq 1$, $x \neq n$.
~~But~~ So $1 < x < n$, since x is a factor different from 1 and n. Then also, n has a factor $y = n/x$. $y \neq n$ and $y \neq 1$, since otherwise $xy \neq n$.

P(n) $\quad$ n

~~Every natural number greater than 1~~ has a prime factorization

$(\forall n \in \mathbb{N}, \{P(\overset{2}{\cancel{0}}), \ldots, P(n-1)\} \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Now by IH x and y have prime factorizations.

ie $x = p_1 \times \cdots \times p_k$ and $y = q_1 \times \cdots \times q_i$

So, $n = xy = p_1 \times \cdots \times p_k \times q_1 \times \cdots \times q_i$ has a prime factorization

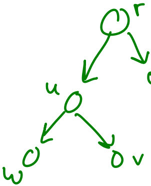
Then, since in both possible cases, n has a prime factorization, we conclude n has a prime factorization, that is $P(n)$

Since n was arbitrary, this shows that $\forall n \in \mathbb{N} - \{0, 1\}, \{P(2), \ldots, P(n-1)\} \Rightarrow P(n)$.

Conclude $\forall n \in \mathbb{N} - \{0, 1\}$, n has a prime factorization

Trees

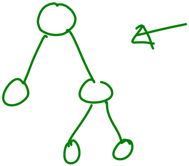
definitions, page 32

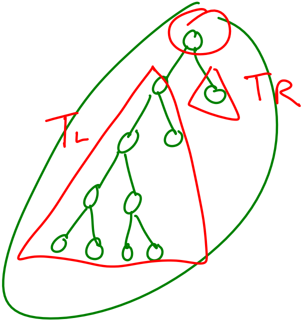


- A tree is a directed graph
- A non-empty tree has a root node, r, such that there is exactly one path from r to any other node.
- If a tree has an edge (u, v), then u is v's parent, v is u's child.
- Two nodes with the same parent are called siblings.
- A node with no children is called a leaf. A non-leaf is called an internal node.
- Binary trees have nodes with ≤ 2 children, and children are labelled either left or right.
- Internal nodes of full binary trees have 2 children.

Tree examples

know your trees. . .





Every full binary tree, except the zero tree, has an odd number of nodes

$(\forall n \in \mathbb{N}, \{P(\overset{1}{\cancel{0}}), \ldots, P(n-1)\} \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Assume n is a natural number greater than 0, and that every FBT with $0 < i < n$ ~~trees~~ nodes has an odd number of nodes.

Every full binary tree, except the zero tree, has an odd number of nodes

$(\forall n \in \mathbb{N}, \{P(0), \ldots, P(n-1)\} \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$