CSC236 fall 2012

complete induction

Danny Heap
heap@cs.toronto.edu
BA4270 (behind elevators)
http://www.cdf.toronto.edu/~heap/236/F12/
416-978-5899

Using Introduction to the Theory of Computation, Section 1.3

Outline

Principle of complete induction

Examples of complete induction

Complete Induction

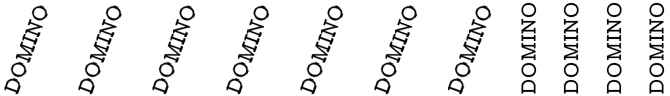
another flavour needed

Every natural number greater than 1 has a prime factorization

Try some examples

How does the factorization of 8 help with the factorization of 9?

More dominoes



$$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$$

If all the previous cases always implies the current case
then all cases are true

Every natural number greater than 1 has a prime factorization

$(\forall n \in \mathbb{N}, \langle P(2), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Every natural number greater than 1 has a prime factorization

$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

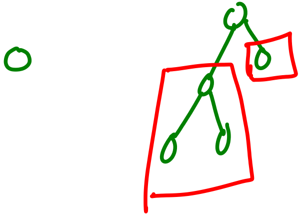
Trees

definitions, page 32

- A tree is a directed graph
- A non-empty tree has a root node, r, such that there is exactly one path from r to any other node.
- If a tree has an edge (u, v), then u is v's parent, v is u's child.
- Two nodes with the same parent are called siblings.
- A node with no children is called a leaf. A non-leaf is called an internal node.
- Binary trees have nodes with ≤ 2 children, and children are labelled either left or right.
- Internal nodes of full binary trees have 2 children.

Tree examples

know your trees. . .



P(n): Every full binary tree, ~~except the zero tree,~~ with n nodes has ~~an~~ n odd. ~~number of nodes~~

$(\forall n \in \mathbb{N}-\{0\}, \langle P(1), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}-\{0\}, P(n)$ ⟵

Assume $n \in \mathbb{N}-\{0\}$ and that for any FBT with i nodes, $0 < i < n$, i is odd.

Suppose T is a FBT with n nodes. There are two possibilities

Case 1, n is 1: Then T has n = 1, an odd number of nodes!

Case 2, n > 1: Then the root is an internal node (there are more nodes) so it has 2 children (FBT).

Notice that the roots children are each roots of non-empty FBT (why?), call them T_L and T_R, with j_L and j_R nodes, respectively,

So $n = j_L + j_R + 1$.

Computer Science
UNIVERSITY OF TORONTO

Every full binary tree, except the zero tree, has an odd number of nodes

$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Since T_L + T_R each have at least 1 node (root has 2 children) j_L and $j_R > 0$. Since +ve number $j_L, j_R, 1$ sum to n, $j_L, j_R < n$. So, by IH, since $0 < j_L, j_R < n$, we know j_L, j_R are odd so $n = \underbrace{j_L + j_R}_{\text{even}} + 1$ odd!

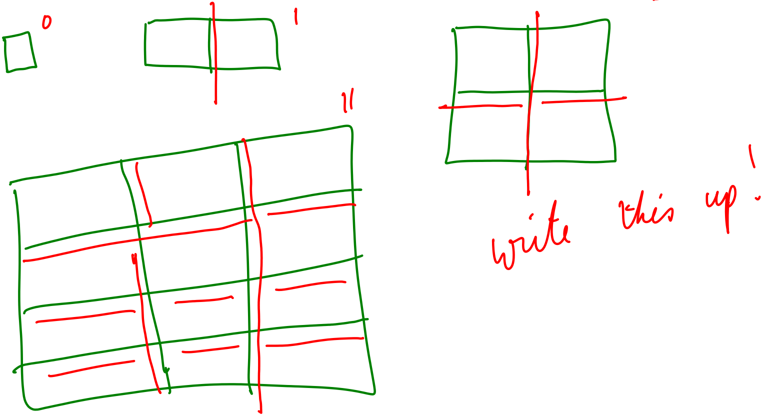
Since n is odd in both possible cases, n is, well, odd!

So $\forall n \in \mathbb{N} - \{0\}$, if every FBT with $0 < i < n$ nodes has i odd, then every FBT with n nodes has n odd.

Conclude $\forall n \in \mathbb{N} - \{0\}$, ~~P~~ $P(n)$.

Every rectangular array of chocolate $m \times n$ squares can be broken up with ? "breaks"

$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$



Every rectangular array of chocolate $m \times n$ squares can be broken up with ? "breaks"

$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Every rectangular array of chocolate $m \times n$ squares can be broken up with ? "breaks"

$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1)\rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$

After a certain natural number n, every postage can be made up by combining 3– and 5– cent stamps

0 ✓
1 ✗
2 ✗
3 ✓
4 ✗
5 ✓
6 ✓
7 ✗

---

8 ✓
9 ✓
10 ✓
11 ✓
12 ✓
13 ✓
14 ✓
15 ✓

write up as MI and CI !