CSC236 fall 2012

regular languages, regular expressions

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Using Introduction to the Theory of Computation, Chapter 7

Outline

regular expressions, regular languages

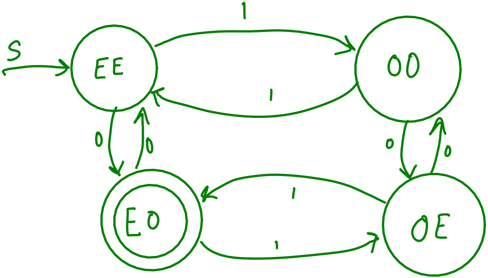
notes

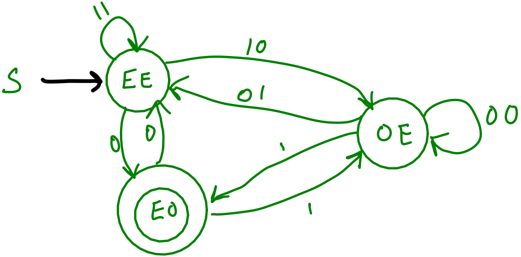
they're equivalent:

$L = L(M)$ for some DFSA M $\Leftrightarrow$ $L = L(M')$ for some NFSA M' $\Leftrightarrow$
$L = L(R)$ for some regular expression R

step 1: convert $L(M)$ to $L(R)$, eliminate states

Even 1s, odd length





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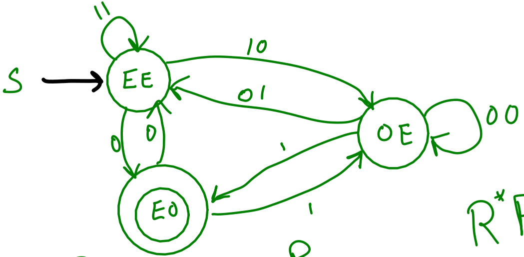
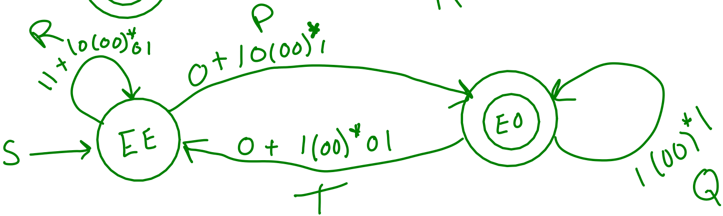


$$R^*P(Q+TR^*P)^*$$

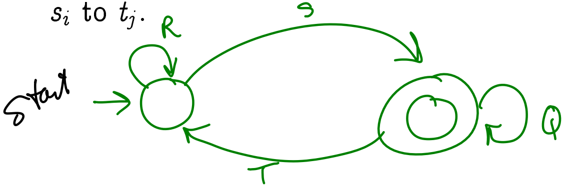


equivalence...

state elimination recipe for state q

1. $s_1 \ldots s_m$ are states with transitions **to** q, with labels $S_1 \ldots S_m$
2. $t_1 \ldots t_n$ are states with transitions **from** q, with labels $T_1 \ldots T_n$
3. Q is any self-loop on q
4. Eliminate q, and add (union) transition label $S_i Q^* T_j$ from s_i to t_j.

$R^* S(Q + TR^*S)^*$



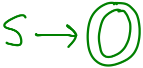
equivalence:

step 2: convert $L(R)$ to $L(M)$:

start with $\emptyset$, ε, $a \in \Sigma$

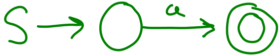
$L = L(R) \Longrightarrow L = L(M)$

$M_\emptyset$



M_ε

M_a, $a \in \Sigma$

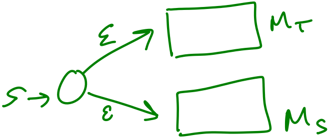


equivalence...

step 2.5: convert $L(R)$ to $L(M)$:
union, concatenation, stars

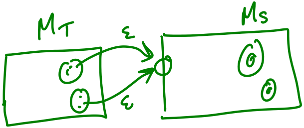
Assume we have M_T, M_S, accept $L(T)$ and $L(S)$

$M_{(T+S)}$



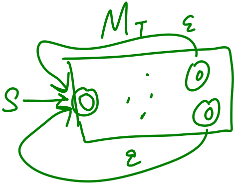
$(0+1)^*0(0+1)(0+1)(0+1)$

$M(TS)$



$(0+1)(000)^*$

$M(T^*)$



notes $L = \{1^n0^n \mid n \in \mathbb{N}\}$

$L = \{\varepsilon, 10, 1100, 111000, \dots\}$

1^*0^*

$(11)^*(00)^*$

$(10)^*$ $\quad \varepsilon, 10, 1010,$

Prof (contradiction)
That no DFSA accepts L.

Suppose M accepts L.
Then M has finite # of states, $k \in \mathbb{N}^+$
What happens with $1^{k+1}0^{k+1} = s$ $\quad 1^k0^k$, $\underline{1^{k+j}0^k}$
in process s, M proceeds

$$\underbrace{q_0 \xrightarrow{1} q_1 \to q_2 \to \cdots \xrightarrow{1} q_{k+1}}_{} \xrightarrow{0} q_{k+2} \to \cdots \to q_{2k+2}$$

Some q_j visited twice.

$$s \to q_j \,(j)^* \to q_{k+2} \cdots \to q_{2k+2}$$

notes

$L = \{ s \in \{0,1\}^* \mid |s| \text{ is prime} \}$

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Colin

Norman

Feyyaz