

Ad-posted by section (Lila, Glin, ...). T2 spoile : CSC236 fall 2012 B overage, bock Fridey - automata and languages FSA

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Using Introduction to the Theory of Computation, Chapter 7

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formal languages

FSAs

notes



some definitions

Many problems can be reduced to languages: logical formulas, identifiers for compilation, natural language recognition. Key question is recognition:  $P^{Voccessing}$ 

Given language L and string s, is  $s \in L$ ?

Languages may be described either by descriptive generators (for example, regular expressions) or procedurally (e.g. finite state automata)

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## more notation

string length: denoted |s|, is the number of symbols in s, e.g. |bba| = 3.

s = t: if and only if |s| = |t|, and  $s_i = t_i$  for  $1 \le i \le |s|$ .

 $s^{R}$ : reversal of s is obtained by reversing symbols of s, e.g.  $1011^{R} = 1101$ .  $\xi^{R} = \xi$   $\xi^{R} = \xi$ 

st or  $s \circ t$ : contcatenation of s and t — all characters of s followed by all those of t, e.g.  $bba \circ bb = bb abb$ .

> $s^k$ : denotes s concatenated with itself k times. E.g.,  $ab^3 = ababab$ ,  $101^0 = \varepsilon$ .

 $\Sigma^{n}: \text{ all strings of length } n \text{ over } \Sigma, \Sigma^{*} \text{ denotes all}$   $\mathfrak{T}^{o} = \{ \mathfrak{E} \}$ 

language operations  

$$5^{*}$$
  $\sum_{L}^{\infty}$   $\sum_{L}^{L}$   $\sim L$   $\sum_{I}^{\infty}$   $\sum_{L}^{I}$   $\sim L$ 

 $\overline{L}$ : Complement of L, i.e.  $\Sigma^* - L$ . If L is language of strings over  $\{0, 1\}$  that start with 0, then  $\overline{L}$  is the language of strings that begin with 1 plus the empty string.

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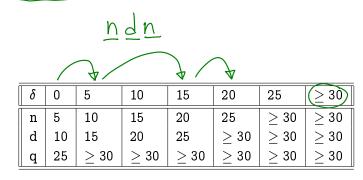
$$L \cup L'$$
: union  $\{2a, b, ba\} = L'$ 

 $L \cap L'$ : intersection

L - L': difference  $L \setminus L'$ 

## states needed to classify a string

what state is a stingy vending machine in based on coins? accepts only nickles (a), dimes (b), and quarters (c), no change given, and everything costs 30 cents useful toy (you'll need JRE)



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build an automaton with formalities... quintuple:  $(Q, \Sigma, q_0, F, \delta)$ Q is set of states,  $\Sigma$  is finite, non-empty alphabet,  $q_0$  is start state F is set of accepting states, and  $\delta: Q \times \Sigma \mapsto Q$  is transition function  $e_q$ .  $\{04, 54, 104, 154, 204, 254, 2304\}$ 

We can extend  $\delta: Q \times \Sigma \mapsto Q$  to a transition function that tells us what state a string takes the automaton to:

$$\delta^* : Q \times \Sigma^* \mapsto Q \qquad \delta^*(q, s) = \begin{cases} \delta(\delta^*(5, n \downarrow), n) \\ = \delta(\delta(\delta^*(5, n), \downarrow), n) \\ = \delta(\delta(\delta^*(5, \epsilon), n), \downarrow), n) \\ q & \text{if } s = \epsilon \\ \delta(\delta^*(q, s'), a) & \text{if } s' \in \Sigma^*, a \in \Sigma \\ \leq -\delta^*q \end{cases}$$

String s is accepted if and only iff  $\delta^*(q_0, s) \in F$ , it is rejected otherwise.

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## example — an odd machine

devise a machine that accepts strings over  $\{a, b\}$  with an odd number of as

Formal proof requires inductive proof of invariant:

$$\delta^*(E,s) = egin{cases} E & ext{if $s$ has even number of $a$s} \ O & ext{if $s$ has odd number of $a$s} \end{cases}$$



## notes

