T2: back before 8 pm, average B
A2: some sections done released soon.
A3: up tomorrow night.

tutorials changed polarity due to fall break ... so suddenly evening section leads off on formal languages... it'll be okay.

CSC236 fall 2012

automata and languages

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Using Introduction to the Theory of Computation, Chapter 7

Outline

formal languages

FSAs

notes

some definitions

alphabet: finite, non-empty set of symbols, e.g. $\{a, b\}$ or $\{0, 1, -1\}$. Conventionally denoted Σ.

$\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \dots\}$

string: finite (including empty) sequence of symbols over an alphabet: abba is a string over $\{a, b\}$. Σ

Λ ϵ Convention: ε is the empty string, never an allowed symbol, Σ^* is set of all strings over Σ.

language: Subset of Σ^* for some alphabet Σ. Possibly empty, possibly infinite subset. E.g. $\{\}$, $\emptyset$
$\{aa, aaa, aaaa, \overset{\epsilon}{\dots}\}$.

N.B.: $\{\} \neq \{\varepsilon\}$.

Many problems can be reduced to languages: logical formulas, identifiers for compilation, natural language processing. Key question is recognition:

Given language L and string s, is $s \in L$?

Languages may be described either by descriptive generators (for example, regular expressions) or procedurally (e.g. finite state automata)

more notation

$\{1, 11\}$ 11

string length: denoted $|s|$, is the number of symbols in s, e.g. $|bba| = 3$.

$s = t$: if and only if $|s| = |t|$, and $s_i = t_i$ for $1 \leq i \leq |s|$.

s^R: reversal of s is obtained by reversing symbols of s, e.g. $1011^R = 1101$.

st or $s \circ t$: contcatenation of s and t — all characters of s followed by all those of t, e.g. $bba \circ bb = bbabb$.

s^k: denotes s concatenated with itself k times. E.g., $ab^3 = ababab$, $101^0 = \varepsilon$.

Σ^n: all strings of length n over Σ, Σ^* denotes all strings over Σ.

language operations

$\overline{L}$: Complement of L, i.e. $\Sigma^* - L$. If L is language of strings over $\{0,1\}$ that start with 0, then $\overline{L}$ is the language of strings that begin with 1 plus the empty string.

$L \cup L'$: union

$L \cap L'$: intersection

$L = \{a, ab\}$
$L' \{0, 1\}$

$L - L'$: difference
$L \setminus L'$

$LL' = \{a0, ab0, a1, ab1\}$

$\mathrm{Rev}(L)$: $= \{s^R : s \in L\}$
$L' \circ L$

concatenation: LL' or $L \cdot L' = \{rt | r \in L, t \in L'\}$. Special cases $L\{\varepsilon\} = L = \{\varepsilon\}L$, and $L\{\} = \{\} = \{\}L$.

more language operations

$$\{\}^0 = \{\varepsilon\}$$

exponentiation: L^k is concatenation of L k times. Special case, $L^0 = \{\varepsilon\}$, including $L = \{\}$!

$$\Sigma^* = \{a\}^*$$

Kleene star: $L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$

$$\{\varepsilon\}^2 = \{\varepsilon\}$$

states needed to classify a string

what state is a stingy vending machine in based on coins?
accepts only nickles (a), dimes (b), and quarters (c),
no change given, and everything costs 30 cents
useful toy (you'll need JRE)

$\Sigma = \{n, d, q\}$

$\underline{n}\,\underline{d}\,\underline{n}$

δ	0	5	10	15	20	25	≥ 30
n	5	10	15	20	25	≥ 30	≥ 30
d	10	15	20	25	≥ 30	≥ 30	≥ 30
q	25	≥ 30	≥ 30	≥ 30	≥ 30	≥ 30	≥ 30

build an automaton with formalities...

quintuple: $(Q, \Sigma, q_0, F, \delta)$

Q is set of states, Σ is finite, non-empty alphabet, q_0 is start state

F is set of accepting states, and $\delta : \underline{Q \times \Sigma} \mapsto Q$ is transition function

We can extend $\delta : Q \times \Sigma \mapsto Q$ to a transition function that tells us what state a **string** s takes the automaton to:

ndn

$\delta^*(q_0, ndn) = \delta(\delta^*(q_0, nd), n)$

$= \delta(\delta(\delta^*(q_0, n), d), n)$

$\delta(\delta(\delta(\underbrace{\delta^*(q_0, \varepsilon)}_{q_0}, n), d), n)$

q_5 15 20

$$\delta^* : Q \times \Sigma^* \mapsto Q \qquad \delta^*(q, s) = \begin{cases} q & \text{if } s = \varepsilon \\ \delta(\delta^*(q, s'), a) & \text{if } s' \in \Sigma^*, a \in \Sigma, s = \end{cases}$$

$s = s'a$

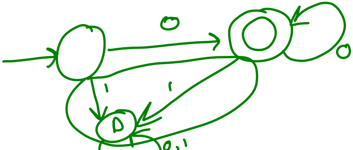
$a \in \Sigma \quad s' \in \Sigma^*$

String s is accepted if and only iff $\underline{\delta^*(q_0, s)} \in F$, it is rejected otherwise.

example — an odd machine

devise a machine that accepts strings over $\{a, b\}$ with an odd number of as

$\Sigma = \{0, 1\}$



L has even # of as + odd length

Formal proof requires inductive proof of invariant:

E + O are states

$$\delta^*(E, s) = \begin{cases} E & \text{if } s \text{ has even number of } a\text{s} \\ O & \text{if } s \text{ has odd number of } a\text{s} \end{cases}$$

for all $k \in \mathbb{N}$, $s \in \Sigma^k$, then $\delta^*(E, S)$ is true. want to conclude that M accept S iff s has ~~even~~ odd # of as.

float machine

$L_1 = \{0, \ldots, 9\}$

$L_2 = \{+, -\}, L_3 = \{.\}$

$L_F = \{s \in L_2^j L_1^m L_3^k L_1^n \mid j, k \leq 1, m, n \geq 1\}$

Devise a machine that accepts L_F