CSC236 fall 2012

Theory of computation

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Course web page 416-978-5899

Using Introduction to the Theory of Computation, Section 1.2

Outline

Why reason about computing?

- It's more than just hacking
- Testing isn't enough
- You might get to like it (?!*)

How to reason about computing

- It's messy...
- It's art...

How to do well at this course

- Read the course information sheet as a two-way promise
- Question, answer, record, synthesize
- Collaborate with respect

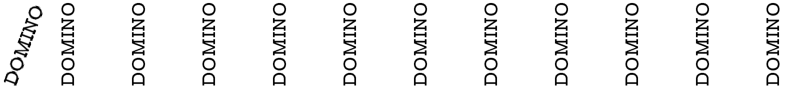
What should you already know?

- Chapter 0 material from *Introduction to Theory of Computation*
- CSC165 material, especially the mathematical prerequisites (Chapter 1.5), proof techniques (Chapter 3), and the introduction to big-Oh (Chapter 4).
- But you can *relax* the structure

What'll you know by December?

- Understand, and use, several flavours of induction
- Complexity and correctness of programs — both recursive and iterative
- Formal languages, regular languages, regular expressions

Domino fates foretold



$$[\, P(0) \;\wedge\; (\forall n \in \mathbb{N}, P(n) \Rightarrow P(n+1)\,)\,] \Longrightarrow \forall n \in \mathbb{N}, P(n)$$

If the initial case works,
and each case that works implies its successor works,
then all cases work

Every set with n elements has exactly 2^n subsets

Use: $[\,P(0) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow P(n+1))\,] \Longrightarrow \forall n \in \mathbb{N}, P(n)$

Scratch work:

Every set with n elements has exactly 2^n subsets...

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For every $n \in \mathbb{N}$, $12^n - 1$ is a multiple of 11

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Scratch work: How to connect n to $n+1$?

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How many base cases do we need?

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How many odd-sized subsets of a set of size n?

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What's $P(n)$?

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Notes