CSC236, Fall 2012
Assignment 3

These problems are to give you some practice with correctness proofs, and regular languages. Start early, and seek out your instructor and teaching assistant when you have questions.

Submit your solutions as a PDF file called `a3.pdf`. You must generate the PDF from a word processor, or LaTeX — no scanned handwritten work will be accepted.

1. Let $L = \{x \in \{0,1\}^* \mid \text{fourth-last symbol in } x \text{ is } 0\}$. Prove that any DFSA that accepts L has at least 16 states. **Hint:** Consider the sixteen binary strings of length four, and what happens if two of them drive a DFSA to the same state.

2. Prove that the following terminates, given the precondition $x \in \mathbb{N}$:

```
y = x * x
while not y == 0 :
  x = x - 1
  y = y - 2 * x - 1
```

   Hint: Trace through the code for a few small values of x, then derive (and prove) a loop invariant that helps prove termination.

3. Design a DFSA that accepts the language of binary strings over $\{0,1\}$ that have a multiple of 4 1s. Devise, and prove a state invariant, and explain how it shows that your DFSA accepts this language.

4. Design an iterative binary search algorithm that is correct with respect to the following precondition/postcondition pair:

   Precondition: A has elements that are comparable with x, $|A| = n > 0$, and A is sorted in non-decreasing order.

   Postcondition: binSearch(x, A) terminates and returns an index p that satisfies:

$$\begin{aligned} A[0 \ldots p] &\leq x < A[p+1 \ldots n-1] \\ -1 &\leq p \leq n-1 \end{aligned}$$

   Prove that if the precondition is satisfied, then your algorithm terminates and satisfies the postcondition. **Hint:** Use the approach from lecture (no need to provide the pictures) where you develop a loop invariant as you write the code.