# CSC236, Fall 2012
# Assignment 1

These problems aim to give you some practice writing proofs of facts from different domains, using induction. Unless you find them easy, you should start working on them early, and be sure to talk them over with your instructor and teaching assistant.

Submit your solutions as a PDF file called `a1.pdf`. You must generate the PDF from a word processor, or LaTeX — no scanned handwritten work will be accepted.

1. A **ternary tree** is a tree where each node has no more than 3 children, and the **height** of a tree is defined as the number of nodes in the longest path[1] from the root to any leaf. Use Complete Induction to prove that if a[2] ternary tree has height $n$, it has no more than $3^n - 2$ nodes.

2. Exponential growth is, typically, faster than polynomial growth, so you should expect that beyond the first few natural numbers $n^3 + n < 3^n$. Determine what the "first few" natural numbers are, and prove that this inequality holds for all other natural numbers, using Mathematical Induction.

3. A set of 3 elements has exactly one subset of size 3 (we'll call it a 3-subset), namely itself. Experiment until you find a formula for the number of 3-subsets that a set of $n + 3$ elements has, then use Mathematical Induction to prove that your formula works for any natural number $n$. You may use, without proof, that a set with $n + 2$ elements has $[(n + 2)(n + 1)]/2$ subsets of size 2.

4. Use Complete Induction or Mathematical Induction[3] to prove that any binary string that begins and ends with the same bit has an even number of occurrences of substrings from $\{01, 10\}$, e.g. 010 has two: 01 and 10. You may find it useful to combine this claim with a similar claim about binary strings that begin and end with different bits, and then prove the combined claims simultaneously.

[1] Some mathematicians define tree **height** to be the number of edges in a longest path.
[2] non-empty — the claim is false for the empty tree
[3] Several other proof techniques will also work