

UNIVERSITY OF TORONTO FACULTY OF ARTS AND SCIENCE

Term test #2

CSC 165H1 DURATION - 50 MINUTES

PLEASE HANDIN AIDS ALLOWED: 8.5" x11" HANDWRITTEN AID SHEET

LAST NAME:

FIRST NAME:

DO NOT turn this page until you have received the signal to start. (In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). When you receive the signal to start, please make sure that your copy of the test is complete.

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

## QUESTION 1. [10 MARKS]

Prove that every natural number that has remainder 5 when divided by 7 has a square that has remainder 4 when divided by 7. Use the proof structure from this course.

## QUESTION 2. [10 MARKS]

Use induction on n to prove that for every natural number n greater than 6,  $2^n > 2n^2$ . You may find it useful to recall that  $(n + 1)^2 = n^2 + 2n + 1$ .

## QUESTION 3. [8 MARKS]

Define the ceiling of a real number x by:

$$\lceil x 
ceil \in \mathbb{Z} \land \lceil x 
ceil \geq x \land ( orall z \in \mathbb{Z}, z \geq x \Rightarrow z \geq \lceil x 
ceil)$$

Use this definition, and the proof structure from this course, to prove  $\forall x \in \mathbb{R}, 3\lceil x \rceil < 3x + 3$ 

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

# 1: \_\_\_\_/10 # 2: \_\_\_\_/10 # 3: \_\_\_\_/ 8

TOTAL: \_\_\_\_/28