PLEASE HAND IN

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UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE

TERM TEST #1

CSC 165H1
DURATION — 50 MINUTES

AIDS ALLOWED: 8.5" x 11" HANDWRITTEN AID SHEET, BOTH SIDES

LAST NAME: Heap

FIRST NAME: Danny

Do NOT *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

QUESTION 1. [22 MARKS]

Consider the following python functions and definitions, where L is a list and P is a boolean function. NOTE: feel free to ask about how python works, since you are being tested on logic, not programming.

```
def q1(L, P) : return False not in [P(x) for x in L]

def q2(L, P) : return False in [P(x) for x in L]

def q3(L, P) : return True not in [P(x) for x in L]

def q4(L, P) : return True in [P(x) for x in L]

L1 = [0, 1, 2, 3, 4, 5]

def P1(x) : return x < 2

def P2(x) : return x < 3

def P3(x) : return (not P2(x)) or P1(x)
```

PART (A) [6 MARKS]

Write the name of each function q1--q4 beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. $\forall x \in L, P(x)$ — q1
2. $\exists x \in L, P(x)$ — q4
3. $\forall x \in L, \neg P(x)$ — q3
4. $\exists x \in L, \neg P(x)$ — q2

mutual negations

mutual negations

PART (B) [16 MARKS]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

P1 : $x < 2$

P3 : $x < 3 \Rightarrow x < 2$

1. q1(L1, P1) False, P1(3) is False, so $\forall x \in L1$, P1(x) is False

2. q2(L1, P1) True, P1(3) is False, so $\exists x \in L1, \neg P1(x)$ is True

3. q3(L1, P1) False, P1(1) is True, so $\forall x \in L1, \neg P1(x)$ is False

4. q4(L1, P1) True — this is the negation of q3

5. q1(L1, P3) False. $2 < 3 \wedge \neg(2 < 2)$, so $\forall x \in L1$, P3(x) $\Rightarrow$ P2(x) is False

6. q2(L1, P3) True — this is the negation of q1

7. q3(L1, P3) False, $4 < 3 \Rightarrow 4 < 2$ is true (vacuous), so $\forall x \in L1, \neg(x < 3 \Rightarrow x < 2)$ is False

8. q4(L1, P3) True — negation of q3

QUESTION 2. [10 MARKS]

PART (A) [5 MARKS]

Consider the following symbolic statement:

$$S1: \quad \forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon$$

1. Write the negation of the symbolic statement $S1$, in such a way that the negation symbol $\neg$ applies only to predicates such as $x > \delta$ or $x^3 > \varepsilon$.

$\exists \varepsilon \in \mathbb{R}^+, \forall \delta \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \wedge \neg(x^3 > \varepsilon)$ either ok.
$\Updownarrow$
$x^3 \leq \varepsilon$

2. Which is true, statement $S1$ or its negation? Briefly explain your reasoning.

~~S1 is true~~. Given $\varepsilon > 0$, pick $\delta = \sqrt[3]{\varepsilon}$. Then
$\forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > (\sqrt[3]{\varepsilon})^3 = \varepsilon$
(so its negation is false)

PART (B) [5 MARKS]

Now the consider the symbolic statement:

$$S2: \quad \exists \delta \in \mathbb{R}^+, \forall \varepsilon \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon$$

1. Write the negation of the symbolic statement $S2$, in such a way that the negation symbol $\neg$ applies only to predicates such as $x > \delta$ or $x^3 > \varepsilon$.

$\forall \delta \in \mathbb{R}^+, \exists \varepsilon \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \wedge \neg(x^3 > \varepsilon)$ either ok
$\Updownarrow$
$x^3 \leq \varepsilon$

2. Which is true, statement $S2$ or its negation? Briefly explain your reasoning.

$\neg S2$ is true. Given $\delta > 0$, pick $\varepsilon = 8\delta^3$ and $x = 2\delta$. Then $x = 2\delta > \delta \wedge x^3 = 8\delta^3 \leq \varepsilon = 8\delta^3$.

QUESTION 3. [6 MARKS]

Suppose F is the set of functions, $D(f)$ means "f is differentiable," and $C(f)$ means "f is continuous." Consider the following statement:

$S3$: "Every function is not differentiable unless it is continuous."

Write the contrapositive, converse, and the negation of $S3$ symbolically. NOTE: in this course we translate "unless" as "if not".

contrapositive $\forall f \in F,\ D(f) \Rightarrow C(f)$

converse $\forall f \in F,\ \neg D(f) \Rightarrow \neg C(f)$

negation $\exists f \in F,\ \neg C(f) \wedge D(f)$.

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

1: _____/22

2: _____/10

3: _____/ 6

TOTAL: _____/38