Prove or disprove each of the following statements. Write detailed proof structures and justify your work.

1. For all natural numbers $n$, if there is a natural number $k$ such that $n=3 k+1$, then there is a natural number $j$ such that $n^{2}=3 j+1$.
2. For all real numbers $r, s$, if $r$ and $s$ are both positive, then $\sqrt{r}+\sqrt{s}=\sqrt{r+s}$.
3. For all real numbers $r, s$, if $r$ and $s$ are both positive, then $\sqrt{r}+\sqrt{s} \neq \sqrt{r+s}$.
4. For all real numbers $x$ and $y, x^{4}+x^{3} y-x y^{3}-y^{4}=0$ exactly when $x= \pm y$.
