# CSC 165 WINTER 2013 TUTORIAL 3

### 1. PROVING EQUIVALENCE

Suppose P, Q, R, and S are statements.

- (1) Prove that  $P \Rightarrow (Q \Rightarrow (R \Rightarrow S))$  is equivalent to  $(P \land Q \land R) \Rightarrow S$ .
- (2) Prove that  $((P \Rightarrow Q) \Rightarrow R) \Rightarrow S$  is equivalent to  $(\neg P \land \neg R) \lor (Q \land \neg R) \lor S$ .

# 2. Negation

Negate the following sentences:

- (1) Every dog has its day, or perhaps its cat.
- (2)  $\forall x \in X, \exists y \in Y, x > y \land y > x$

## 3. GUARANTEES

Consider the statement:

(S1) A and B are both guarantees that C is true.

- (1) Write (S1) symbolically. Use parentheses "(" and ")" to make your answer precise.
- (2) Choose some appropriate phrases to replace A, B and C. Use these to write (S1) in English. Does this cause you to reconsider your answer to (1)?
- (3) Suppose (S1) is true and A is false. What, if anything, can be determined about B and C? Briefly justify.

#### 4. SAMPLE SOLUTIONS

### 1. PROVING EQUIVALENCE

(1) Prove that  $P \Rightarrow (Q \Rightarrow (R \Rightarrow S))$  is equivalent to  $(P \land Q \land R) \Rightarrow S$ .

$$P \Rightarrow (Q \Rightarrow (R \Rightarrow S)) \quad \Leftrightarrow \quad \neg P \lor (\neg Q \lor (\neg R \lor S)) \qquad [\text{transform} \Rightarrow \text{to} \neg \text{ and } \lor] \\ \Leftrightarrow \quad (\neg P \lor \neg Q \lor \neg R) \lor S \qquad [\text{associativity of } \lor] \\ \Leftrightarrow \quad \neg (P \land Q \land R) \lor S \qquad [\text{DeMorgan's Law}] \\ \Leftrightarrow \quad (P \land Q \land R) \Rightarrow S \qquad [\text{transform} \neg \text{ and } \lor \text{ to } \Rightarrow]. \end{cases}$$

$$(2) \text{ Prove that } ((P \Rightarrow Q) \Rightarrow R) \Rightarrow S \text{ is equivalent to } (\neg P \land \neg R) \lor (Q \land \neg R) \lor S. \\ ((P \Rightarrow Q) \Rightarrow R) \Rightarrow S \quad \Leftrightarrow \quad \neg (\neg (\neg P \lor Q) \lor R) \lor S \qquad [\text{transform} \Rightarrow \text{ to} \neg \text{ and } \lor]$$

$$((1 \rightarrow Q) \rightarrow R) \rightarrow S \quad \Leftrightarrow \quad (((1 \rightarrow Q) \wedge R) \vee S) \quad [\text{transform} \rightarrow to + and \vee Q) \\ \Leftrightarrow \quad ((\neg P \lor Q) \land \neg R) \lor S \quad [\text{DeMorgan's Law}] \\ \Leftrightarrow \quad (\neg P \land \neg R) \lor (Q \land \neg R) \lor S \quad [\text{distributivity of } \land]$$

### 5. Negation

- Every dog has its day, or perhaps its cat.
   Sample solution: Some dog has neither its day nor its cat.
- (2)  $\forall x \in X, \exists y \in Y, x > y \land y > x$ Sample solution:  $\exists x \in X, \forall y \in Y, x \leq y \lor y \leq x$

## 2. Guarantees

- (1)  $(A \Rightarrow C) \land (B \Rightarrow C)$  or  $(A \lor B) \Rightarrow C$
- (2) "Being rich and being beautiful are both guarantees that one is hated."
- (3) Suppose (S1) is true and A is false. What, if anything, can be determined about B and C? Briefly justify. Nothing. It tells us nothing about C, and A is unrelated to B.
- (4) Suppose (S1) is true and C is false. What, if anything, can be determined about A and B? Briefly justify. A is false and B is false. This comes from the contrapositive(s) of the implication(s), which must be true.