

CSC165 winter 2013

Mathematical expression

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Course notes, chapter 4



Outline

notes

Prove $3n^2 + 2n \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2\}$



Special case? what happens if you add a constant?

Prove $3n^2 + 2n + 5 \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2\}$



Look at the leading term

Prove: $7n^6 - 5n^4 + 2n^3 \in \mathcal{O}(6n^8 - 4n^5 + n^2)$

Use $\mathcal{O}(6n^8 - 4n^5 + n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq c(6n^8 - 4n^5 + n^2)\}$



how to prove $n^3 \notin \mathcal{O}(3n^2)$?

Negate $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow n^3 \leq c3n^2\}$



non-polynomials

Big-oh statements about polynomials are pretty easy to prove:
 $f \in \mathcal{O}(g)$ exactly when the highest-degree term of g is no smaller than the highest-degree term of f .

What about functions such as $\log(n)$ or 3^n ? Logarithmic functions are in big-Oh of **any** polynomial, whereas exponential functions (with a base bigger than 1) are not in big-Oh of any polynomial. How do you prove such things?

Prove $2^n \notin \mathcal{O}(n^2)$

Use $\lim_{n \rightarrow \infty} 2^n / n^2$

Do you know anything about the ratio $2^n / n^2$, as n gets very large? How do you evaluate:

$$\lim_{n \rightarrow \infty} \frac{2^n}{n^2}$$

If the limit evaluates to ∞ , then that's the same as saying:

$$\forall c \in \mathbb{R}^+, \exists n' \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n' \Rightarrow \frac{2^n}{n^2} > c$$

Once your enemy hands you a c , you can choose an n' with the required property.

Notes