A1 - results on MarkUs T1 - Friday CSC1

CSC165 winter 2013

Mathematical expression

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Course notes, chapter 3





Outline

non-boolean functions

notes

non-boolean functions

Take care when expressing a proof about a function that returns a non-boolean value, such as a number:

$$\lfloor x \rfloor$$
 is the largest integer $\leq x$.

Now prove the following statement (notice that we quantify over $x \in \mathbb{R}$, not $\lfloor x \rfloor \in \mathbb{R}$:

$$\forall x \in \mathbb{R}, |x| < x+1$$



using more of the definition

You may have been disappointed that the last proof used only part of the definition of floor. Here's a symbolic re-writing of the definition of floor:

$$\forall x \in \mathbb{R} \qquad y = \lfloor x \rfloor \Leftrightarrow y \in \mathbb{Z} \land y \leq x \land (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$$

The full version of the definition should prove useful to prove:

$$orall x \in \mathbb{R}, \lfloor x
floor > x-1$$



proving something false

Define a sequence:

$$orall n \in \mathbb{N} \qquad a_n = \lfloor n/2
floor$$

(of course, if you treat "/" as integer division, there's no need to take the floor. Now consider the claim:

$$\exists i \in \mathbb{N}, orall j \in \mathbb{N}, j > i \Rightarrow a_j = a_i$$

The claim is false. Disprove it.





proof by cases

Sometimes your argument has to split to take into account possible properties of your generic element:

$$\forall n \in \mathbb{N}, n^2 + n \text{ is even}$$

A natural approach is to factor $n^2 + n$ as n(n + 1), and then consider the case where n is odd, then the case where n is even.



proof about limits
$$\mathbb{R}^{+}$$
, $\forall y \in \mathbb{R}^{+}$, $|y - \pi| < \delta \Rightarrow |y^{2} - \pi^{2}| < \varepsilon$

In proving this claim you can't control the value of ε or y, but you can craft δ to make things work out.

$$orall arepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, orall y \in \mathbb{R}, |y-\pi| < \delta \Rightarrow |y^2-\pi^2| < arepsilon$$

The claim is true. The proof format should be already familiar to you. A good approach is to fill in as much as possible, leaving the actual value of δ out until you have more intuition about it.

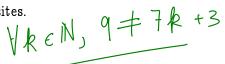


use uniqueness

Suppose you have a predicate of the natural numbers:

$$\forall n \in \mathbb{N}$$
 $N = 3$
 $S(n) \Leftrightarrow \exists k \in \mathbb{N}, n = 7k + 3$

Is $S(3 \times 3)$ true? How do you prove that? It's useful to check out the remainder theorem from the sheet of mathematical prerequisites.





get wrong right

Be careful proving a claim false. Consider the claim, for some suitably defined X, Y and P, Q:

$$S: \quad \forall x \in X, \forall y \in Y, P(x,y) \Rightarrow Q(x,y)$$

To disprove S, should you prove:

$$\forall x \in X, \forall y \in Y, P(x,y) \Rightarrow \neg Q(x,y)$$

What about

$$\forall x \in X, \forall y \in Y, \neg (P(x, y) \Rightarrow Q(x, y))$$

Explain why, or why not.





Define T(n) by:

$$\forall n \in \mathbb{N}$$
 $T(n) \Leftrightarrow \exists i \in \mathbb{N}, n = 7i + 1.$

Take some scrap paper, don't write your name on it, and fill in as much of the proof of the following claim as possible:

$$orall n \in \mathbb{N}, \, T(n) \Rightarrow \, T(n^2)$$

Now fill in as much of the **disproof** of the following claim as possible:

$$\forall n \in \mathbb{N}, T(n^2) \Rightarrow T(n)$$



Notes

VielN, FjelN, j> i Najta;

assume i is a typical retural number # in order to # introduce of Pick j = 1+2. Then jein # since i, z ein Then j = i+2 > i # add i to both sides of 2>0

Then j = i+2 > i # add i to both sides of 2>0 1003 Also $a_j = \begin{bmatrix} 3/2 \end{bmatrix} = \begin{bmatrix} \frac{j+2}{2} \end{bmatrix} = \begin{bmatrix} \frac{j}{2} + 1 \end{bmatrix}$ > 1/2 # LX > X-1 > li/2] # defn of [] Then $a_j > a_i$, is $a_j \neq a_i$ = a_i Then $j > i \land a_i \neq a_i$ # conjunction of above.

Then $j > i \land a_i \neq a_i$ # exhibited j = i + 2Then $j \in \mathbb{N}$, $j > i \land a_j \neq a_i$ # exhibited j = i + 2

Conclude VieIN, BjeN, j>i / aj #a;

Notes 1. Prove: $\forall \varepsilon \in \mathbb{R}^{+}, \exists s \in \mathbb{R}^{+}, \forall y \in \mathbb{R}, |y-7| < \delta \Rightarrow |y^{2}-7|^{2} | < \epsilon$ assume $\xi \in \mathbb{R}^{+}$ # figure out lader

Pick $S = \frac{1}{2}$ # figure out lader

assume $|y - \pi| < S$ Then $|y - \pi|^{2} = \dots = |(y - \pi)(y + \pi)| = |y - \pi|^{2}$ Then $|y^2, \eta^2| \le \xi$ Then $|y-7| < \delta \Rightarrow |y^2, \eta^2| < \xi \# \Rightarrow$ Then \ y e R, |y. 71 < \(\exists = \) | \(y^2 - 7^2 \) < \(\exists \frac{\pm}{\pm} \) who Then ISER, ... # introduced I Conclude $\forall \epsilon e R^{\dagger}, --- + in troduces \forall computer Science UNIVERSITY OF TORONTO$