CSC165 winter 2013 Mathematical expression

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Course notes, chapter 3

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## Outline

non-boolean functions

notes



Take care when expressing a proof about a function that returns a non-boolean value, such as a number:

Now prove the following statement (notice that we quantify over  $x \in \mathbb{R}$ , not  $\lfloor x \rfloor \in \mathbb{R}$ :

Do not 
$$\forall x \in \mathbb{R}, \lfloor x \rfloor < x + 1$$
  
 $\forall \perp x \rfloor \in \mathbb{R}, \dots$ 

x is the largest integer < x.

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## using more of the definition Assume

You may have been disappointed that the last proof used only part of the definition of floor. Here's a symbolic re-writing of the definition of floor:

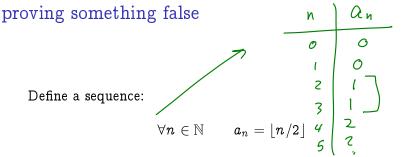
$$orall x \in \mathbb{R} \qquad y = \lfloor x 
floor \Leftrightarrow y \in \mathbb{Z} \land y \leq x \land iggl( orall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y iggr)$$

The full version of the definition should prove useful to prove:

$$egin{aligned} & \bigvee x \in \mathbb{R}, \lfloor x 
floor > x-1 \end{aligned}$$

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(of course, if you treat "/" as integer division, there's no need to take the floor. Now consider the claim:

$$\exists i \in \mathbb{N}, orall j \in \mathbb{N}, j > i \Rightarrow a_j = a_i$$

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The claim is false. Disprove it.  $\forall i \in \mathbb{N}, \exists j \in \mathbb{N}, j > i \land \alpha_i \neq q_i$  Sometimes your argument has to split to take into account possible properties of your generic element:

$$orall n \in \mathbb{N}, n^2 + n$$
 is even

A natural approach is to factor  $n^2 + n$  as n(n + 1), and then consider the case where n is odd, then the case where n is even.



assume  $\chi$  is a generic real number # in order  $[\chi] \leq \chi$  # by definition In proving this claim you can't control the value of  $\varepsilon$  or y, but you can craft  $\delta$  to make things work out. The claim is true. The proof format should be already familiar to you. A good approach is to fillin as much as possible, leaving the actual value of  $\delta$  out until you have more intuition about it.

Conclude, 
$$\forall x \in \mathbb{R}$$
,  $\lfloor x \rfloor < x + 1$  # introduced.  $\forall$ 

proof allout mills E R # in order to introduce V Then LxJE Z # from def. Then  $\lfloor x \rfloor + | \in \mathbb{Z} \ \# \ closure, since \ Lx \rfloor, I \\ \# \ \in \mathbb{Z}_{+}$ In proving this claim you can't control the value of  $\varepsilon$  or y, but you can craft  $\beta$  to make things work out. # contropositive !  $\forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall y \in \mathbb{R}, |y - \pi| < \delta_{y} \Rightarrow |y^2 - \pi_{\varepsilon}^2| \not\in \varepsilon$ The claim is true. The proof format should be already familiar to you. A good approach is to fill in as much as possible, leaving the actual value of  $\delta$  out until you have more intuition about it.  $\dot{x} > \chi - 1$ Conclude VXER, LXJ>X-1