SLOGs - 132 possibly submitted... others needed RSN.

- Tutorial tomorrow

CSC165 winter 2013

Mathematical expression

- office Hour: Wednesday 2-4
- Help Centre Mon-Thurs 4-6
- TA office hours, next week TBA.

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Course notes, chapter 2

Outline

leftovers...

Notes

"natural" language

- minority we won't

unless $\equiv$ if - not

Translate "unless" as "if not":

Don't knock it unless you've tried it.

? ?

Don't knock if not you've tried it.

If not you tried it, then don't knock it.

$\underbrace{\text{you tried it}}_{T}$ $\underbrace{\text{knock it}}_{K}$

$\neg T \Rightarrow \neg K \equiv K \Rightarrow T$

idiom

D = persons
P = vegetarians
Q – salad eaters.

example that distinguishes ① from ②

Some expressions for restricting domains are more common than others.

$\forall x \in D,\ x \in P \cap Q$

- "Every D that is a P is also a Q." Usually
① $\forall x \in D, P(x) \Rightarrow Q(x)$. Less common $\forall x \in D \cap P, Q(x)$.
What about $\forall x \in D, P(x) \wedge Q(x)$ ($\wedge$ means "and")?
②

"and"

some thing

- "Some D that is a P is also a Q." Usually
① $\exists x \in D, P(x) \wedge Q(x)$. Less common $\exists x \in D \cap P \cap Q$.
What about $\exists x \in D, P(x) \Rightarrow Q(x)$?
②

$D = \mathbb{N}$
P = solutions to $x^2 = 5$
$Q = \{x \mid x < x+5\}$

$7 \in D$, if $7^2 = 5$ then $7 < 12$.

conjunction: $\wedge$

Combine two statements by claiming they are both true with logical "and":

$A(x)$ and $B(x)$ (python keyword **and** works like this)

$A(x) \wedge B(x)$ ($\wedge$ is a symbol for "and")

As sets: $x \in A \cap B$

Notice that a conjunction is **false** if either part is false. "The employee makes less than 100,000 and more than 60,000," is true for Gwen, but false for Ellen.

$G \in L \cap M$

Employee	Gender	Salary
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000
Gwen	female	95,000

watch out for English "and"

Sometimes the English word "and" is used to smear some meaning over several components:

There is a pen and a telephone.

In the universe of objects, O, with predicates $P(x)$ (x is a pen) and $T(x)$ (x is a telephone), you could try to translate this as $\exists x \in O, P(x) \wedge T(x)$. What's a better translation into symbols?

Occasionally English usage of **and** will differ from logical usage even in mathematical material:

The solutions are $x < 10$ *and* $x > 20$

The solutions are $x < 20$ *and* $x > 10$

The first statements probably meant the union of the two sets, or the logical **or**. The second meant the intersection, so the logical **and** is appropriate.

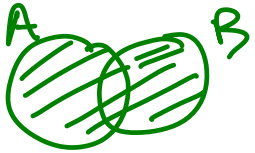
disjunction: $\vee$

Combine two statements by claiming that at least one of them is true using **or** ($\vee$ in symbols).

$A(x)$ or $B(x)$ (the python keyword or works like this)

$A(x) \vee B(x)$ (in symbols)

$x \in A \cup B$ (as sets)



Notice the close connection between the symbols for conjunction and intersection, $\wedge$, $\cap$, and the symbols for disjunction and union, $\vee$, $\cup$. Coincidence? In any case, you may use it as a mnemonic.

"The employee is female or earns more than 35,000."

Employee	Gender	Salary
Betty	female	500
Carlos	male	40,000
Doug	male	30,000
Ellen	female	50,000
Flo	female	20,000
[illegible]	[illegible]	35,000

silly English tricks

In logic we use **or** generously, or inclusively, to mean something like "and/or". Sometimes we convey the **inclusive or** by saying something like "A or B, or both." Be aware that natural English sometimes uses or to mean "A or B, but not both" — something we'd call **exclusive or** in logic:

> *Either we play the game my way, or I'm taking my ball and going home.*

not inclusive!

negation: $\neg$

$\exists e \in E, O(e) \wedge \neg F(e)$

Negate the statement "All employees earning over 110,000 are female." Usually prepending the word "Not" will work, and in logic we use the corresponding symbol $\neg$:

$\wedge$

$$\neg(\forall e \in E, O(e) \Rightarrow F(e))$$

A good exercise is to "work" the negation $\neg$ as far into the statement as possible. The statement is true exactly when its negation is false.

The original statement is universally quantified, so it says something about an absence of counterexamples. The negation of the original statement should claim something about the presence of counterexamples.

special negation idiom

Negating implications is a common task. There are several equivalent ways of doing this, but some are more common than others. Try negating the following in such a way that the $\neg$ symbol applies to the "smallest possible" part of the expression:

$$\forall x \in X, P(x) \Rightarrow Q(x)$$

Now for symmetry, negate the following in such a way that the $\neg$ symbol applies to the "smallest possible" part of the expression:

$$\exists x \in X, P(x) \wedge \neg Q(x)$$

standard negation

Negated expressions have some standard transformations:

- $\neg\forall x \in X, \ldots \Leftrightarrow \exists x \in X, \neg \ldots$
- $\neg\exists x \in X, \ldots \Leftrightarrow \forall x \in X, \neg \ldots$
- $\neg(P(x) \Rightarrow Q(x)) \Leftrightarrow P(x) \wedge \neg Q(x)$
- $\neg(P(x) \wedge Q(x)) \Leftrightarrow P(x) \Rightarrow \neg Q(x)$ (has this become asymmetrical?)

Push the $\neg$ symbol "as far in" to the following expression as possible:

$$\neg(\forall x \in X, \exists y \in Y, P(x) \Rightarrow Q(x))$$

Notes