CSC165 review

April 5, 2013

(POSSIBLE) TOPICS YOU SHOULD BE ON TOP OF

- quantifiers $\forall, \exists$, possibly mixed
- implication $\Rightarrow$
- negation of complex statements
- venn diagrams/truth tables
- connectives $\neg, \wedge, \vee$, manipulating tem
- proof techniques (direct proof of implication, contrapositive, contradiction)
- proof structure (assume $\Rightarrow$ indent), comments
- algorithm analysis (various size of steps okay)
- worst-case complexity of algorithms
- big-Oh, big-Omega, big-Theta without using limit techniques
- using limit techniques
- computability, e.g. halt

REVIEW MATERIALS

- tutorial exercises and solutions
- assignments and solutions
- lecture examples
- course notes

SPARSE COLLECTION OF EXAMPLES

1. Consider the code for matrix multiplication below, where A, B, and C are two-dimensional arrays. Assume that A, B, and C are square ($n \times n$) arrays, and that each operation that doesn't depend on n costs 1 "step." Compute an over-estimate of the number of steps required by the algorithm that is bounded above by some constant multiple of n^3, and explain your estimate. Compute an under-estimate of the number of steps required by the algorithm that is bounded below by some constant multiple of n^3, and explain your estimate. What do your computations tell you about this algorithm?

```
MatrixMultiplication(A, B)
1.    if (A.cols != B.rows)
2.       return null
3.    i = 0
4.    while (i < A.rows){
5.       k = 0
6.       while (k < B.cols){
7.          C[i][k] = 0
8.          j = 0
9.          while (j < A.cols){
10.            temp = C[i][k]
11.            result = A[i][j] · B[j][k]
12.            C[i][k] = temp + result
13.            j = j + 1
               }
14.         k = k + 1
         }
15.      i = i + 1
      }
16.   return C
```

2. Define $U(n) : \exists k \in \mathbb{N}, n = 7k + 2$, and $V(n) : \exists k \in \mathbb{N}, n = 7k + 4$. Use the formal proof structure from this course to prove that for all natural numbers n, $U(n) \Rightarrow V(n^2)$. Is the converse true? Prove (or disprove) the converse, using the formal proof structure from this course.

3. Let $f(n) = 3n + 7n^3$, and let $g(n) = 17 + 34n^2$. Use the formal proof structure from this course to prove that $g \in O(f)$, and that $f \notin O(g)$.

4. Some graffiti in Robarts Library claims that you won't finish an undergraduate degree unless you sell your soul. In order to test the claim, the world's population is divided into four groups:

   - People who have sold their souls.
   - People who haven't finished an undergraduate degree.
   - People who have not sold their souls.
   - People who have finished an undergraduate degree.

   Which of the four groups must be questioned, and which can be safely ignored? Explain.

Exam tactics

The exam is nine questions long, marked out of 72, and lasts three hours. It is comprehensive, that is you are responsible for the entire twelve-week semester. Some questions are similar to material you've worked on for assignments, term tests, or tutorials. You always have the option of leaving a question blank or writing "I do not know how to answer this" for 20% of the marks applicable to that portion of the exam. For a formal proof, you will receive roughly half marks if you write a correct outline and do not write any steps that you can't justify.

Here are some suggestions for the best use of your time.

1. Make sure you understand what you're being asked to do before you begin writing. When in doubt, ask me or the other invigilator a question and we will try to provide a fair answer.

2. Write the outline of a proof, even if there are steps you can fill in. Indicate which steps you can't fill in. Specify which things you assume without proof.

3. Use the spaces left on the exam paper as an indication of expected length. There will be some extra blank pages at the end of the exam.