CSC165 winter 2013 Mathematical expression

Danny Heap heap@cs.toronto.edu BA4270 (behind elevators) http://www.cdf.toronto.edu/~heap/165/W13/ 416-978-5899

Course notes, chapter 4 (big-oh, big-S2, big-O)

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Bounded below

some theorems

notes



non-polynomials

Big-oh statements about polynomials are pretty easy to prove: $f \in \mathcal{O}(g)$ exactly when the highest-degree term of g is no smaller than the highest-degree term of f.

What about functions such as log(n) or 3^n ? Logarithmic functions are in big-Oh of any polynomial, whereas exponential functions (with a base bigger than 1) are not in big-Oh of any polynomial. How do you prove such things?

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 $\operatorname{Prove}\ 2^n
ot\in \mathcal{O}(n^2)$ Use $\lim_{n o\infty} 2^n/n^2$

Do you know anything about the ratio $2^n/n^2$, as n gets very large? How do you evaluate:

$$\lim_{n\to\infty}\frac{2^n}{n^2}\ =\ \infty$$

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If the limit evaluates to ∞ , then that's the same as saying:

$$\begin{array}{c} \forall c \in \mathbb{R}^+, \exists n' \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n' \Rightarrow \frac{2^n}{n^2} > c \\ \text{no notion body sign right} \end{array}$$

Once your enemy hand's you a c, you can choose an n' with the required property.

JBEN, VneN, n≥B⇒ 2ª≤ci2 prove $2^n \not\in \mathcal{O}$ R^{*} ϵN , $\exists n \in N$, $n \ge B \land 2^n >$ cR', VB Using l'Hôpital's rule and limits # 6 R assume B assume \mathcal{O} lim In2. ln2. 24 Then . s, 3 n'ε IN, meaning of by n > # e NjB,n'EN. B + nn B. By choice of n # ລ", 1,2 $2^{"} > Cn^{2}$. So



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big-Omega

Prove
$$f(n) = n^2$$
, $g(n) = 15n^2 + 5$, then $f \in SZ(g)$

Notice that the definition of big-Omega differs in just one character from big-Oh:

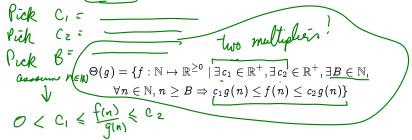
If you're proving $f \in \Omega(g)$, you get to choose c and B to suit your proof. Notice that it would be really unfair to allow c to be zero.

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one more bound

 $f(n) = 15n^2 + 5$ $g(n) = n^2$

It often happens that functions are bounded above and below by the same function. In other words, $f \in \mathcal{O}(G) \land f \in \Omega(g)$. We combine these two concepts into $f \in \Theta(g)$.



You might want to draw pictures, and conjecture about appropriate values of c_1 , c_2 , B for $f = 5n^2 + 15$ and $g = n^2$.

How to prove general statement about two functions: $(f \in \mathcal{O}(g) \land g \in \mathcal{O}(h)) \Rightarrow f \in \mathcal{O}(h)$



how about: $f \in \mathcal{O}(g) \Rightarrow g \in \Omega(f)$



prove or disprove: $f \in \mathcal{O}(g) \Rightarrow f \cdot f \in \mathcal{O}(g \cdot g)$

prove or disprove: $(f \in \mathcal{O}(h) \land g \in \mathcal{O}(h)) \Rightarrow (f + g) \in \mathcal{O}(h)$

prove or disprove: $f \in \mathcal{O}(g) \Rightarrow f \in \mathcal{O}(g \cdot g)$



Notes Pick
$$c_{i} = \frac{15}{20}$$
 then $c_{i} \in \mathbb{R}^{4}$
Pick $c_{z} = \frac{20}{1}$ then $c_{z} \in \mathbb{R}^{4}$
Pick $B = \frac{1}{1}$ then $B \in \mathbb{N}$.
Cassume $n \in \mathbb{N}$ # to intro U .
Cassume $n \geq B$. # to intro U .
Chen $f(n) = 16n^{2} + 6$
 $\leq 15n^{2} + 5h^{2} = 20h^{2}$ # $n \geq 1$
 $\leq 15h^{2} + 5h^{2} = 20h^{2}$ # $n \geq 1$
 $\leq c_{z} n^{2}$ # $C_{z} = 20$
Cheo $f(n) = 15h^{2} + 5$
 $\geq 15h^{2}$ # omit the term
 $= c_{z} n^{2}$ # $C_{i} = 15$

Computer Science UNIVERSITY OF TORONTO