CSC165, Winter 2013 Assignment 2 Due Wednesday March 6th, 11:59 p.m.

The aim of this assignment is for you to practice structuring and expressing some proofs. You may work in groups of no more than three students, and you should produce a single solution in a PDF file named a2.pdf, submitted to MarkUs. You must produce a2.pdf by **typing** your solutions and processing the typed input into a PDF with your favourite word processor or typesetting program. Handwritten and then scanned PDF files are huge and often illegible.

You will receive 20% of the marks for any question you either leave blank, or write "I cannot answer this." You will receive substantial parts marks if you present the outline of a proof/disproof with clear indication of the steps you are unable to complete.

You must decide whether each of the statements below is true or untrue, and then prove or disprove, as appropriate. Your proof/disproof should follow the structure used in this course, that is indentation and justification of steps. Please feel free to use either precise English or the logical symbols taught in this course (or both) in your proofs.

NB: If you cannot decide whether a statement is true or false, write out the proof structure of **both** the proof and disproof, clearly indicating the remaining steps you are unable to justify. These outlines will earn you significant part marks, whereas you will receive **no credit** (and that's significantly less than if you leave it blank) if you either "prove" a false statement to be true, or "disprove" a true statement. You have a couple of weeks to think over whether each statement is true or false.

- 1. Prove or disprove: $\forall x, y \in \mathbb{R}, x < y \Rightarrow (\exists z \in \mathbb{R}, x < z \land z < y).$
- 2. Prove or disprove: $\forall m, n \in \mathbb{N}, m < n \Rightarrow (\exists k \in \mathbb{N}, m < k \land k < n)$
- 3. Prove or disprove: If n is a natural number that has $n^2 \mod 11 = 3$, then $n \mod 11 = 5$. Your proof/disproof must use the definition of a mod b on page 11 of the Course Notes.
- 4. Prove or disprove: If n is a natural number that has $n \mod 11 = 5$, then $n^2 \mod 11 = 3$. Your proof/disproof must use the definition of a mod b on page 11 of the Course Notes.
- 5. Prove or disprove: For all quadruples of positive real numbers w, x, y, z, If w/x < y/z then:

$$\left(rac{w}{x} < rac{w+y}{x+z}
ight) \wedge \left(rac{w+y}{x+z} < rac{y}{z}
ight)$$

Hint: The material on inequalities on page 12 of the Course Notes may be helpful.

6. Prove or disprove: For every pair of positive natural numbers (m, n), if $m \ge n$, then the gcd(m, n) = gcd(n, m - n). Your proof/disproof must use the definition of the gcd (Greatest Common Divisor) on page 12 of the Course Notes.