## CSC165, Winter 2013 Assignment 1 Due January 30th, 11:59 p.m.

The aim of this assignment is for you to practice using quantifiers, implications, logical connectives and expressions. You may work in groups of no more than three students, and you should produce a single solution in a PDF file named a1.pdf, submitted to MarkUs.

You will receive 20% of the marks for any question you either leave blank, or write "I cannot answer this."

- 1. Suppose A is the set of aphorisms, D(h) means h is dodecahedral, and C(h) means h is catalytic (you needn't worry about the meaning of aphorism, dodecahedral, or catalytic for the rest of this question). Write the negation of each of the following sentences in English and in symbolic form.
  - (a) Every aphorism is catalytic unless it is dodecahedral.
  - (b) No aphorism is both catalytic and dodecahedral.
  - (c) All aphorisms that are not catalytic are dodecahedral.
  - (d) Some dodecahedral aphorisms are catalytic.
  - (e) Every aphorism is catalytic if, and only if, it is dodecahedral.
- 2. Consider the sentence:

S: For all triples of natural numbers m, n, p, if p is prime and p divides mn, then p divides m or p divides n.

Each of the statements below is equivalent to either the converse, the contrapositive, or the negation of S. You must decide which label fits each statement, and explain your thinking.

- (a) For some triples of natural numbers m, n, p, neither m nor n is divisible by p, yet p is prime and p divides mn.
- (b) For every triple of natural numbers m, n, p, if p divides neither m nor n, then p is not prime or p doesn't divide mn.
- (c) For every triple of natural numbers m, n, p, if p is not prime or p doesn't divide mn, then p divides neither m nor n.
- (d) There is a triple of natural numbers m, n, p, such that p is prime, p divides mn, and p divides neither m nor n.

- 3. Suppose X is a set that contains developers, and projects. MegaMoth is the name of a project, and Codefinger is the name of one of the developers. Several predicates are defined on X: D(x) means x is a developer, P(x) means x is a project, M(x, y) means x manages y, W(x, y) means x works on y, E(x, y) means x equals y, and I(x, y) means x is more important than y. Use the set X, the predicates and constants above, along with the logical connectives you have learned, to either translate an English sentence into symbolic form, or a symbolic sentence into English.
  - (a) There is exactly one developer in X who is more important than Codefinger
  - (b)  $\forall x \in X, \forall y \in X, \forall z \in X, \forall w \in X, (W(x, y) \land W(z, w) \land I(y, w)) \Rightarrow \neg M(z, x)$
  - $(\mathsf{c}) \ \forall x \in X, (P(x) \land \exists y \in X, (W(y,x) \land (\forall w \in X, W(w,x) \Rightarrow E(w,y)))) \Rightarrow E(x, \texttt{MegaMoth})$
  - $(\texttt{d}) \ \forall x \in X, (\forall y \in X, W(y, \texttt{MegaMoth}) \Rightarrow M(x, y)) \Rightarrow I(x, \texttt{Codefinger})$
  - (e) Codefinger has been a developer on every project in X except MegaMoth.
- 4. Suppose T is a set of natural numbers. Consider the statement:

S2: Every element of T is an integer power of 2.

Which of the following statements imply S2? Which of the following statements are implied by S2? Explain.

- (a) T has at most 1 member that is odd.
- (b) If i and j are elements of T, and j is greater than i, then i divides j.
- (c) The only prime number that divides elements of T is 2.
- (d) T has no elements.
- (e)  $T = \{4, 32, 128\}.$