

PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term test #1

CSC 165H1

Duration — 50 minutes

PLEASE HAND IN

aids allowed: 8.5" x 11" handwritten aid sheet, both sides

Last Name: Sample
First Name: Solution

Do not turn this page until you have received the signal to start.
(In the meantime, please fill out the identification section above,
and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

Question 1. [22 MARKS]

Consider the following python functions and definitions, where S , $S1$ are sets and P , $P1$ – $P4$ are boolean functions. **Note:** feel free to ask about how python works, since you are being tested on logic, not programming.

```
def q0(S, P):
    return all({P(x) for x in S})

def q1(S, P):
    return any({P(x) for x in S})

def q2(S, P):
    return not any({P(x) for x in S})

def q3(S, P):
    return not all({P(x) for x in S})

S1 = {0, 1, 2, 3, 4, 5}

def P1(x):
    return x > 2

def P2(x):
    return x > 3

def P3(x):
    return (not P2(x)) or P1(x)

def P4(x):
    return P1(x) and P2(x)
```

Part (a) [6 MARKS]

Write the name of each function $q0$ – $q3$ beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

-
1. $\forall x \in S, P(x)$ q_0
2. $\exists x \in S, P(x)$ q_1
3. $\forall x \in S, \neg P(x)$ q_2
4. $\exists x \in S, \neg P(x)$ q_3

Part (b) [16 MARKS]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

1. $q0(S1, P2)$ False
 $\forall x \in S1, x > 3$ ——— 0, 1, 2, 3 are counterexamples

2. $q1(S1, P1)$ True
 $\exists x \in S1, x > 2$ ——— 3, 4, 5 are examples.

3. $q2(S1, P2)$ False
 $\forall x \in S1, \neg(x > 3)$ ——— 4, 5 are counterexamples

4. $q3(S1, P1)$ True
 $\exists x \in S1, \neg(x > 2)$ ——— 0, 1, 2 are examples.

5. $q0(S1, P3)$ True
 $\forall x \in S1, \neg(x > 3) \vee (x > 2)$ ——— no counterexamples, since any number is either ≤ 3 or > 2 (or both)

6. $q1(S1, P3)$ True
 $\exists x \in S1, \neg(x > 3) \vee (x > 2)$ — every element of $S1$ is an example

7. $q2(S1, P4)$ False
 $\forall x \in S1, \neg(x > 2 \wedge x > 3)$ — 4, 5 are counterexamples.

8. $q3(S1, P4)$ True
 $\exists x \in S1, \neg(x > 2 \wedge x > 3)$ — 0, 1, 2, 3 are examples.

Question 2. [10 MARKS]**Part (a)** [5 MARKS]

Consider the following symbolic statement:

$$S1: \quad \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \exists q \in \mathbb{N}, m + n = q$$

1. Write the negation of the symbolic statement $S1$, in such a way that the negation symbol \neg applies only to predicates such as $m + n = q$.

② $\exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \forall q \in \mathbb{N}, \neg(m + n = q)$

2. Which is true, statement $S1$ or its negation? Briefly explain your reasoning.

① $S1$ is true.
 ② for any $m, n \in \mathbb{N}$, choose $q = m + n \in \mathbb{N}$.

Part (b) [5 MARKS]

Now the consider the symbolic statement:

$$S2: \quad \exists q \in \mathbb{N}, \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, m + n = q$$

1. Write the negation of the symbolic statement $S2$, in such a way that the negation symbol \neg applies only to predicates such as $m + n = q$.

② $\forall q \in \mathbb{N}, \exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \neg(m + n = q)$

2. Which is true, statement $S2$ or its negation? Briefly explain your reasoning.

① $\neg S2$ is true
 ② for any $q > 0$, pick $m = n = 0$. If $q = 0$,
 pick $m = n = 1$.

Question 3. [6 MARKS]

Come up with an example of sets D , P , and Q so that one of statements $S3$, $S4$ is true, and the other is false. Explain which is true, which false, and why.

Interpret $P(x)$ as $x \in P$, $Q(x)$ as $x \in Q$, and $D(x)$ as $x \in D$.

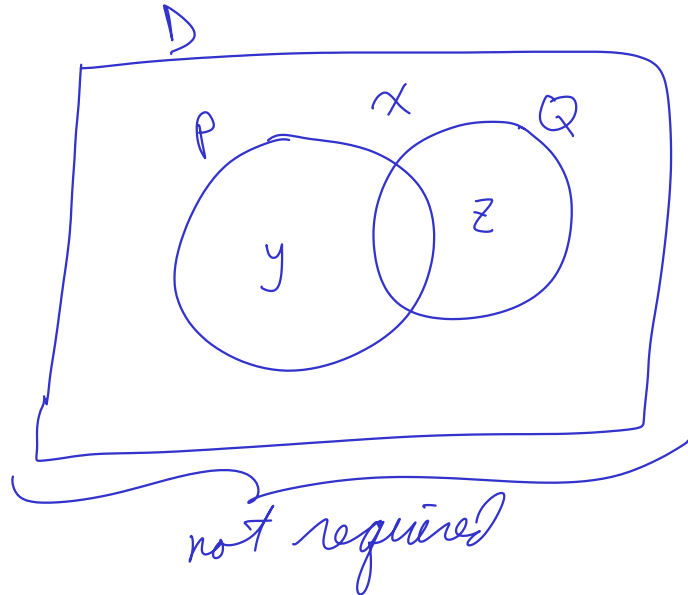
$S3: \exists x \in D, P(x) \Leftrightarrow Q(x)$ *True*

$S4: \exists x \in D, P(x) \wedge Q(x)$ *False*

$$D = \{x, y, z\}$$

$$P = \{y\}$$

$$Q = \{z\}$$



There is an element of D , namely x , which is not in P outside Q nor Q outside P , so $S3$ is true. However, there is no element of D in $P \cap Q$, so $S4$ is false.

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

1: _____/22

2: _____/10

3: _____/ 6

TOTAL: _____/38