

penny piles

Work with 1 or 2 other students, and choose one of your group as the recorder. The recorder should keep a written record of his/her group's discussion of the problem below, which they may use as the basis for a contribution to the problem-solving wiki (see below). Use the following headings to organize the discussion:

Make a start on the problem for about 5–7 minutes, when you should stop and review any choices you've made about how to proceed. I have suggested a couple of approaches to consider on the back of this sheet, but you should only look at these after you have made a good start on the problem. Use only one hint at a time, since they lead to different solution paths.

- Understand the problem.
- Devise a plan.
- Carry out the plan.
- Look back, figure out when and how you're stuck.

You are sitting in front of two drawers. The left drawer contains 64 pennies, the right drawer contains nothing. Can you arrange things so that one of the drawers has 48 pennies, using combinations of the following two operations, *l* and *r*?

- l*: If the left drawer has an even number of pennies, you may transfer half of them to the right drawer. If the left drawer has an odd number of pennies, operation *l* is disallowed.
- r*: If the right drawer has an even number of pennies, you may transfer half of them to the left drawer. If the right drawer has an odd number of pennies, operation *r* is disallowed.

Choose another number in the range $[0, 64]$. Starting from the same initial position, can you arrange things so that one of the drawers has that number of pennies? Are there any numbers in that range that are impossible to achieve? What about starting with a different number of pennies in the left drawer?

You can continue working on this problem at: https://www.cgi.cdf.toronto.edu/~heap/cgi-bin/Solvent/wiki.pl?Problem_Solving_Home_Page/PennyPiles Login as **sleuth**, with password **eureka**.

Hints

Hint 1, work backwards: Imagine you have already carried out steps that give you the desired number of pennies in one drawer. What would the **second-last** step be (the step just before the successful step)?

Hint 2, smaller cases: Is there any connection between the steps to get 24 in a drawer when you start with 64, and getting 12 in a drawer when you start with 32?

Hint 3, draw a picture: Draw a tree diagram of all the possible results (amounts of pennies in each drawer). Try to be systematic.