

Consider the following algorithm:

```
def order(L):
    """ (list of numbers) -> None
        Order L from smallest to largest. L is changed in-place. """
    i = 1
    while i < len(L):
        j = i
        while j > 0 and L[j] < L[j-1]:
            L[j], L[j-1] = L[j-1], L[j] # swap L[j] and L[j-1]
            j = j - 1
        i = i + 1
```

1. Compute the number of “swaps” (executing the line that says swap) performed by the algorithm in the worst-case, on any list L of length n .

The def line can be ignored: it is part of the syntax to define a function, but not something that actually gets executed every time we call the function. So we count only the steps in the body of the function.

The outer loop iterates over $i = 1, 2, 3, \dots, n-1$.

For each value of i , the inner loop iterates over $j = i, i-1, \dots, 2, 1$, *as long as* $L[j] < L[j-1]$. In the worst-case (when L is initially sorted in reverse order), this happens for *every* value of j .

For each value of j , the algorithm swaps once: 1 time when $i = 1$ (for $j = 1$), 2 times when $i = 2$ (for $j = 2$ and $j = 1$), \dots , $n - 1$ times when $i = n-1$ (for $j = n-1$ and \dots and $j = 1$).

So in total, the algorithm performs exactly $1 + 2 + \dots + n - 1 = n(n - 1)/2 = n^2/2 - n/2$ swaps, in the worst-case.

2. Compute the number of “steps” (basic operations) performed by the algorithm in the worst-case, on any list L of length n . Count a step each time a line is visited.

As before, we count only the lines in the body. The outer loop iterates over $i = 1, 2, 3, \dots, n-1$.

For each value of i , the inner loop iterates over $j = i, i-1, \dots, 2, 1$, in the worst-case (as argued in the first question).

For each value of j , the algorithm performs 3 steps. So over all values of j , a total of $3i$ steps.

In addition, for each value of i , there are steps performed outside of the inner loop: 3 steps for the lines outside the inner loop, and an additional 1 step to evaluate the last inner loop condition — when the condition becomes False. So each iteration of the outer loop performs $3i + 4$ steps.

Together with the first line, and the extra 1 step for the last outer loop condition, the number of steps performed by the algorithm is exactly:

$$\begin{aligned} \left(\sum_{i=1}^{n-1} (3i + 4) \right) + 2 &= 3 \left(\sum_{i=1}^{n-1} i \right) + 4 \left(\sum_{i=1}^{n-1} 1 \right) + 2 \\ &= 3n(n-1)/2 + 4(n-1) + 2 \\ &= 3n^2 + 5n - 4 \end{aligned}$$