

1. Write the *complete proof* of each of the following statements. Note that, for the first two statements, we have already written down the proof structures in previous tutorial.

(a) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leq y \Rightarrow \exists z \in \mathbb{Z}, x \leq z \leq y$

(b) $\forall x \in \mathbb{Z}, (\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$

(c) $\neg(\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y > x \wedge a_y > a_x)$ — for the sequence $A = 2, 4, 6, 8, 9, 7, 5, 3, 1, 0, 0, 0, 0, \dots$. You may assume $a_i = 0$ for $i \geq 10$.