

# CSC165 Fall 2014, Assignment #3

Due December 1st, 10:00 p.m.

This assignment gives you some practice with asymptotic proofs (big-Oh, big-Omega, big-Theta), halting problem reductions, and limits. You may work in groups of no more than three students, and you should produce a single solution in a PDF file named a3.pdf, submitted to [MarkUs](#). You must produce a3.pdf by **typing** your solutions and processing the typed input into a PDF with your favourite word processor or typesetting program. Handwritten and then scanned PDF files are huge and often illegible (and not allowed...).

You will receive 20% of the marks for any question you either leave blank, or write “I cannot answer this.” You will receive substantial parts marks if you present the outline of a proof/disproof with clear indication of the steps you are unable to complete. All your proofs must use the course proof structure, including comments. The structure is relaxed in the following ways:

- You may introduce more than one assumption on the same line, for example “Assume  $n$  is a natural number, and that  $n$  is even...”
- You don’t need matching “bookends” for each structural element you introduce. For example, if you introduce an existential with “Pick  $x = 3$ . Then  $x \in \mathbb{R}$  # introduce existential...” you don’t need to match this with “ $\exists x \in \mathbb{R}$ ...” later on.

If you are not sure whether to prove or disprove, write the proof structure, making it clear which steps you cannot justify. You will receive 0 for “justifying” false things.

1. **Prove or disprove:**

$$\forall e \in \mathbb{R}^+, \exists d \in \mathbb{R}^+, \forall x, y \in \mathbb{R}^+, |x - y| > d \Rightarrow |x + y| > e$$

2. **Prove or disprove:**  $6n^3 - 4n^2 + 3n + 2$  is in  $\Omega(5n^3 - n^2 + n + 1)$ .
3. **Prove or disprove:**  $15n^2$  is in  $\Omega(3 \times 2^n)$ . **Hint:** Consider using limit techniques from calculus, including l’Hôpital’s rule as part of this proof. Please talk to your TA/instructor/Help Centre when needed.
4. **Prove or disprove:**  $2^n$  is in  $\mathcal{O}(3^n)$ . **Hint:** Consider using the limit techniques of calculus and notice that

$$\lim_{n \rightarrow \infty} \frac{2^n}{3^n} = \lim_{n \rightarrow \infty} \left(\frac{2}{3}\right)^n$$

5. Given the set  $\mathcal{F} = \{f : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}\}$ , **Prove or disprove:**

$$\forall f, g \in \mathcal{F}, f \in \mathcal{O}(g) \vee f \in \Omega(g)$$

6. Prove that the function `meaning_of_life` below is not computable:

```
def meaning_of_life(f, I) :  
    """  
    Return True if f(I) returns 42, False otherwise.  
    """
```

Emulate the technique from the course notes to reduce `halt` to `meaning_of_life`