Question 1. [5 marks]

Read over the declaration of class **BTNode** as well as the header and docstring for function **has_siblings**. Then complete the implementation of **has_siblings**.

```
 class BTNode:
    """A node in a binary tree."""

    def __init__(self: 'BTNode', item: object,
                 left: 'BTNode' =None, right: 'BTNode' =None) -> None:
        """Initialize this node.
        """
        self.item, self.left, self.right = item, left, right


def has_siblings(T: BTNode, item1: object, item2: object) -> bool:
    """Return True if tree rooted at T has a node with a node.left.item ==
    item1 and node.right.item == item2, otherwise False.

    >>> T = BTNode(1, BTNode(2, BTNode(3)), BTNode(4, BTNode(5), \
BTNode(6)))
    >>> has_siblings(T, 5, 6)
    True
    >>> has_siblings(T, 2, 3)
    False
    """

    if T is None:
        return False
    else:
        return ((T.left and T.left.item == item1 and
                  T.right and T.right.item == item2) or
                 has_siblings(T.left, item1, item2) or
                 has_siblings(T.right, item1, item2))
```

Marking notes: 1 mark for None base case, 2 marks for checking whether T has appropriate pair of children, 2 marks for checking T.left and T.right for appropriate sibling pairs.

E1: misses case where only one child is None

Question 2. [5 marks]

Read over the declarations of classes **BTNode** and **LLNode**, as well as the header and docstring for function **root_to_leaves**. Then implement the function **root_to_leaves**.

```
 class BTNode:
    """A node in a binary tree."""

    def __init__(self: 'BTNode', item: object,
                 left: 'BTNode' =None, right: 'BTNode' =None) -> None:
        """Initialize this node.
        """
        self.item, self.left, self.right = item, left, right


class LLNode:
    """A node in a linked list."""

    def __init__(self: 'LLNode', item: object, link: 'LLNode' =None) -> None:
        """Initialize this node.
        """
        self.item, self.link = item, link

    def __repr__(self: 'LLNode') -> str:
        """Return a string that represents self in constructor (initializer) form.

        >>> b = LLNode(1, LLNode(2, LLNode(3)))
        >>> repr(b)
        'LLNode(1, LLNode(2, LLNode(3)))'
        """
        return ('LLNode({}, {})'.format(repr(self.item), repr(self.link))
                if self.link else 'LLNode({})'.format(repr(self.item)))

    def __eq__(self: 'LLNode', other: 'LLNode') -> bool:
        """Return whether LLNode self is equivalent to LLNode other"""
        return (isinstance(other, LLNode) and
                self.item == other.item and self.link == other.link)

def root_to_leaves(T: BTNode) -> list:
    """
    Return list of paths from T to each of its leaves, or []
    if T is None.  Each path is a linked list formed from LLNodes.
    You should return a list containing a single-node linked list
    when T has no children.

    >>> T = BTNode(1, BTNode(2, None, BTNode(3)), BTNode(4, BTNode(5), BTNode(6)))
    >>> L1 = root_to_leaves(T)
    >>> L2 = [LLNode(1, LLNode(2, LLNode(3))), LLNode(1, LLNode(4, LLNode(5))), \
LLNode(1, LLNode(4, LLNode(6)))]
    >>> len(L1) == len(L2) and all([p in L2 for p in L1])
```

```
    True
    """

    if T is None:
        return []
    elif T.left is None and T.right is None:
        return [(LLNode(T.item,None))]
    else:
        leftchpaths = root_to_leaves(T.left)
        rightchpaths = root_to_leaves(T.right)
        leftpaths = [LLNode(T.item, P) for P in leftchpaths]
        rightpaths = [LLNode(T.item, P) for P in rightchpaths]
        return leftpaths + rightpaths
```

Question 3. [5 marks]

Read over the class declaration for **BTNode** and the header and docstring for function **ordered_and_bounded**. Then implement **ordered_and_bounded**.

```
class BTNode:
    """A node in a binary tree."""

    def __init__(self: 'BTNode', item: object,
                 left: 'BTNode' =None, right: 'BTNode' =None) -> None:
        """Initialize this node.
        """
        self.item, self.left, self.right = item, left, right


def ordered_and_bounded(T: BTNode, lower: int, upper: int) -> list:
    """Return a list of items, in ascending order, from nodes of T,
    with all items no less than lower and no greater than upper.
    Return [] if T is None.  You are *not* allowed to sort any list, and
    you should visit as few nodes as possible.

    preconditions:   -- node items in T are comparable,
                     -- T is a binary search tree in ascending order,
                        that is, all items in every left sub-tree are less
                        than the sub-tree's root and all items in every right
                        sub-tree are more than the sub-tree's root

    >>> T = BTNode(4, BTNode(2, BTNode(1), BTNode(3)) , BTNode(6, \
BTNode(5), BTNode(7)))
    >>> ordered_and_bounded(T, 2, 5)
    [2, 3, 4, 5]
    """

    if T is None:
        return []
    else:
        return ((ordered_and_bounded(T.left, lower, upper)
                 if lower < T.item else []) +
                ([T.item] if lower <= T.item <= upper else []) +
                (ordered_and_bounded(T.right, lower, upper)
                 if upper > T.item else []))
```

Marking notes: 1 mark for None base case. 1 mark for getting list from left subtree if lower ¡= T.item. 1 mark for getting list from right subtree if upper ¿= T.item. 2 marks for adding T.item to list if it is in interval [lower, upper]. 1 mark off if extra nodes are visited, that is BST property not used. 1 mark off if list is sorted.

Question 4. [6 marks]

Read the functions **hybrid_search** and **hybrid_search2**. For each function, decide which of the following complexity classes best describe that function's worst-case performance on a list of **n** elements:

$\mathcal{O}(1)$ $\mathcal{O}(\lg n)$ $\mathcal{O}(n)$ $\mathcal{O}(n \lg n)$ $\mathcal{O}(n^2)$

For each function, explain why your choice of big-Oh complexity makes sense. Also explain what behaviour you expect **hybrid_search** and **hybrid_search2** should exhibit when run on a computer on a list of size **2n** versus a list of size **n**.

```
def hybrid_search(x:int,L:list) -> bool:
    """precondition: L is sorted
    >>> L = [1,5,9, 9, 9, 12, 12, 15, 19,20,40,41,42,43,50,100,500]
    >>> hybrid_search(21,L)
    False
    >>> hybrid_search(100,L)
    True
    """
    def helper(i,j) -> bool:
        # precondition: 0 <= i <= j < len(L)
        if (j-i) < 10:
            return any([y == x for y in L[i:j+1]])
        if x < L[(i+j)//2]:
            return helper(i, (i+j)//2-1)
        elif x > L[(i+j)//2]:
            return helper((i+j)//2+1, j)
        else:
            return True
    return helper(0,len(L)-1)
```

$\mathcal{O}(\lg n)$. The helper method is called approximately $\lg n - 3$ times, and then a linear search of no more than 10 items is performed, so the complexity is proportional to $\lg n$. I expect that the running time would increase by a constant as the size of the input list was doubled.

Marking notes: 2 marks for indicating $\mathcal{O}(\lg n)$ and giving a suitable explanation. 1 mark for indicating that run time would increase by a constant if the length of the input list were doubled.

```
def hybrid_search2(x:int,L:list) -> bool:
    """precondition: L is sorted
    >>> L = [1,5,9, 9, 9, 12, 12, 15, 19,20,40,41,42,43,50,100,500]
    >>> hybrid_search(21,L)
    False
    >>> hybrid_search(100,L)
    True
    """
    def helper(i,j) -> bool:
```

```
        # precondition: 0 <= i <= j < len(L)
        if (j-i) < len(L)/10:
            return any([y == x for y in L[i:j+1]])
        if x < L[(i+j)//2]:
            return helper(i, (i+j)//2-1)
        elif x > L[(i+j)//2]:
            return helper((i+j)//2+1, j)
        else:
            return True
    return helper(0,len(L)-1)
```

O(n). The call to the helper methods occur at most 4 times before $j - i < len(L)/10$, and then we must search a slice of size between n/10 and n/20. Then each element in a slice with at least $n/20$ elements must be inspected. I expect the running time to roughly double if I increase the size of the list from n to $2n$.

Marking notes: 2 marks for choosing O(n) with a suitable explanation. 1 mark if their expectation of how running time scales with doubling the list size is consistent with (whatever) choice of complexity class they make.