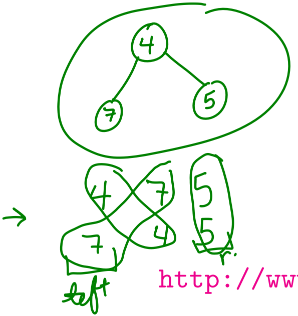
- Welcome to daylight savings!
- A1 — we can re-run unit tests at a small discount...
- A2 — regextree.py now posted
- E3 - due Thurs.

CSC148 winter 2014

BSTs, big-Oh

week 9



Danny Heap
heap@cs.toronto.edu
BA4270 (behind elevators)
http://www.cdf.toronto.edu/~heap/148/W14/
416-978-5899

March 10, 2014

Outline

performance

big-oh

wrapper/node binary tree

instead of single tree class, separate node and bst classes:

```
class BTNode:
    """Binary Tree node."""

    def __init__(self: 'BTNode', data: object,
                 left: 'BTNode'=None,
                 right: 'BTNode'=None) -> None:
        """Create BT node with data, children left and right."""
        self.data, self.left, self.right = data, left, right
```

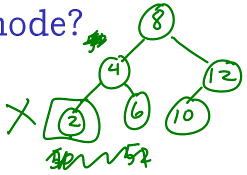
binary search tree

Add a condition: data in left subtree is less than that in the root, which in turn is less than that in right subtree. Now search is more efficient...

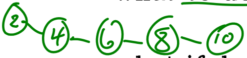
```
class BST:
    """Binary search tree."""

    def __init__(self: 'BST', root: BTNode=None) -> None:
        """Create BST with BTNode root."""
        self._root = root
```

also, possibly, record size other features.

deletion of data from BST rooted at node?



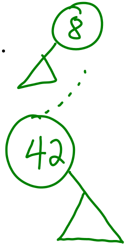
- what return value? — new root!
- what to do if node is None? return



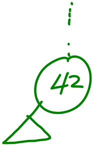
- what if data to delete is less than that at node?

  ↳ delete from left + return this node
- what if it's more?

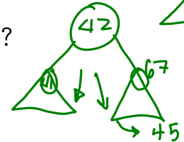
  ↳ delete from right + return this.
- what if the data equals this node's data and...



  - this node has no left child



  - ... no right child?
  - both children?



recall list searching

linear (linear search)
that is if list has length n,
we expects # of steps proportional to n

You've already seen algorithms for seeing whether an element is contained in a `list`:

`[97, 36, 48, 73, 156, 947, 56, 236]`

What is the performance of these algorithms in terms of list size? What about the analogous algorithm for a tree?

binary search $\lg n$ $\log n$

$\log_3 n$

BST efficiency?

Binary search of a list allowed us to ignore (roughly) half the list. Searching a **binary search tree** allows us to ignore the left or right subtree — nearly half in a well-balanced tree.

If we're searching the tree rooted at node n for value v, then one of three situations are possible:

lg n performance for n nodes.

- node n has value v
- v is less than node n's value, so we should search to the left
- v is more than node n's value, so we should search to the right

performance...

We want to measure **algorithm** performance, independent of hardware, programming language, random events

Focus on the **size** of the input, call it n. How does this affect the resources (e.g. processor time) required for the output? If the relationship is linear, our algorithm's complexity is $\mathcal{O}(n)$ — roughy proportional to the input size n.

running time analysis

algorithm's behaviour over large input (size **n**) is common way to compare performance — how does performance vary as **n** changes?

- constant: $c \in \mathbb{R}^+$ (some positive number) → [illegible]
- logarithmic: $c \log n$ → binary search
- linear: cn (probably not the same c) → linear search
- quadratic: cn^2
- cubic: cn^3
- exponential: $c2^n$ →
- horrible: cn^n or $cn!$ → bogosort, matrix

running time analysis

abstract away difference between similar worst-case performance, e.g.

- one algorithm runs in $(0.3365n^2 + 0.17n + 0.32)\mu s$
- another algorithm runs in $(0.47n^2 + 0.08n)\mu s$
- in both cases doubling n quadruples the run time. We say both algorithms are $\mathcal{O}(n^2)$ or "order n^2" or "oh-n-squared" behaviour.

asymptotics

If any reasonable implementation of an algorithm, on any reasonable computer, runs in number of steps **no more than** $cg(n)$ (some constant c), we say the algorithm is $\mathcal{O}(g(n))$. Graphing various examples where $g(n) = n^2$ shows how we ignore the constant c as n gets large (say $7n^2, 2n^2 + 1$ versus $43n + 2, n = 1297$).

case: $\lg n$

this is the number of times you can divide n in half before reaching 1.

- refresher: $a^b = c$ means $\log_a c = b$.
- this runtime behaviour often occurs when we "divide and conquer" a problem (e.g. binary search)
- we usually assume $\lg n$ (log base 2), but the difference is only a constant:

$$2^{\log_2 n} = n = 10^{\log_{10} n} \Longrightarrow \log_2 n = \log_2 10 \times \log_{10} n$$

- so we just say $\mathcal{O}(\lg n)$.

hierarchy

Since big-oh is an **upper-bound** the various classes fit into a hierarchy:

$$\mathcal{O}(1) \subseteq \mathcal{O}(\lg n) \subseteq \mathcal{O}(n) \subseteq \mathcal{O}(n^2) \subseteq \mathcal{O}(n^3) \subseteq \mathcal{O}(2^n) \subseteq \mathcal{O}(n^n)$$