A1 – due tomorrow, 10 pm – see email for office hours
T1 – in 2 weeks, samples of old tests posted, material up to end of today.

CSC148 winter 2014

recursive structures

week 6

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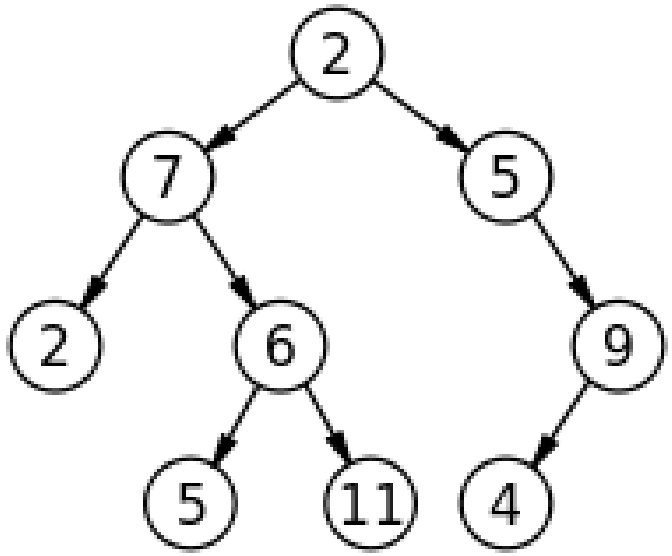
Outline

recursion, natural and otherwise



terminology

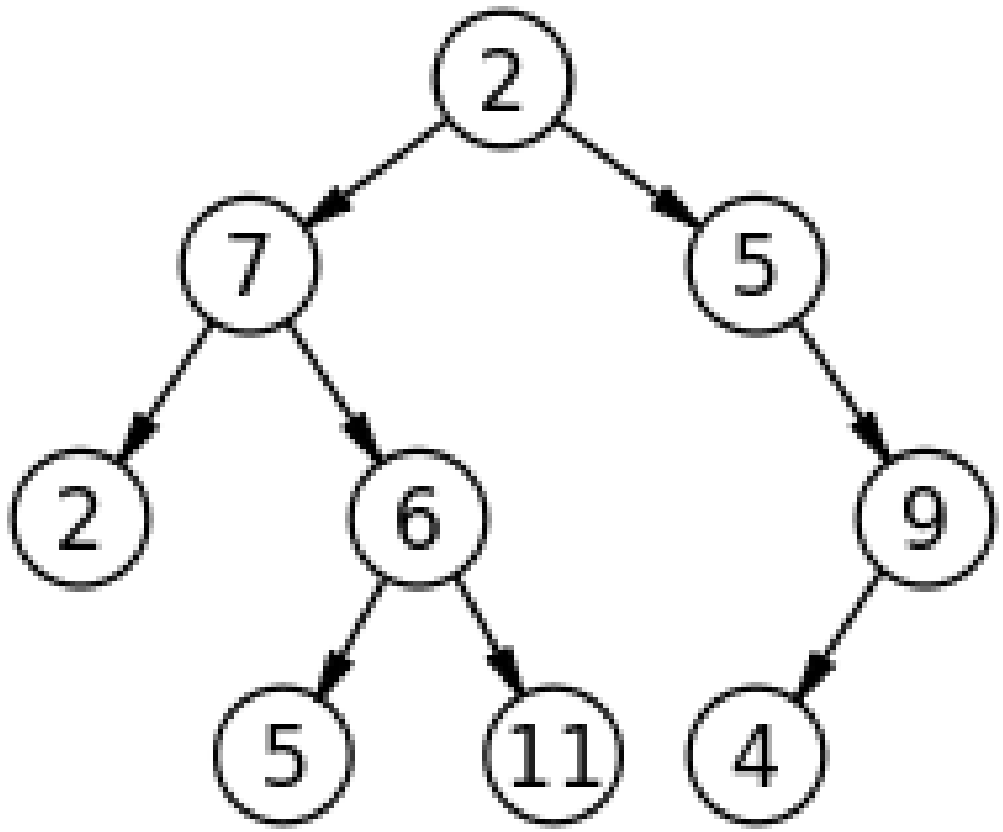
- set of **nodes** (possibly with values or labels), with directed **edges** between some pairs of nodes
- One node is distinguished as **root**
- Each non-root node has exactly one parent.
- A **path** is a sequence of nodes $n_1, n_2, \ldots, n_k$, where there is an edge from n_i to n_{i+1}. The **length** of a path is the number of edges in it
- There is a unique path from the root to each node. In the case of the root itself this is just n_1, if the root is node n_1.
- There are no **cycles** — no paths that form loops.

more terminology

- **leaf**: node with no children
- **internal node**: node with one or more children
- **subtree**: tree formed by any tree node together with its descendants and the edges leading to them.
- **height**: Maximum path length in a tree. A node also defines a height, which is the maximum path length of the tree rooted at that node
- **arity, branching factor**: maximum number of children for any node.

pre-order traversal

Visit root, then pre-order left subtree, then pre-order right subtree



exercise: code for preorder traversal

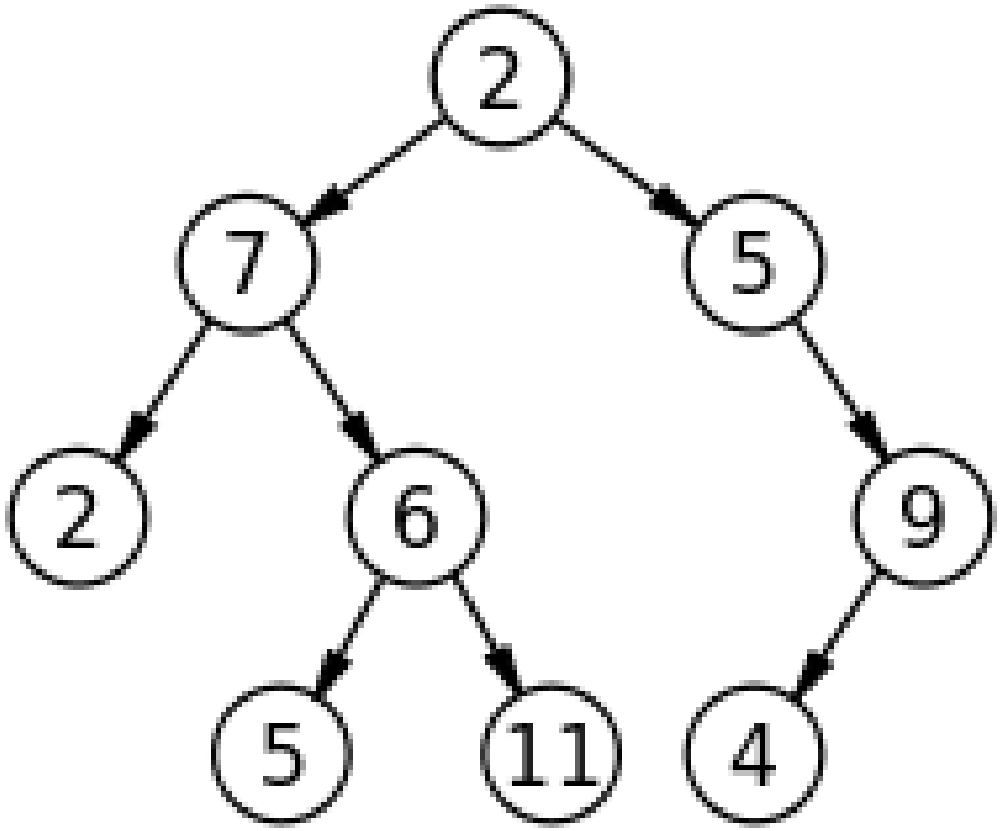
```
"""
A TreeList is either None or a Python list with 3 elements, where
  --- element 0 is a value
  --- element 1 is a TreeList
  --- element 2 is a TreeList
"""

def preorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in preorder

    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
    >>> preorder(T)
    [5, 4, 3, 2, 1]
    """
```

in-order traversal

Visit in-order left subtree, then root, then in-order right subtree



exercise: code for inorder traversal

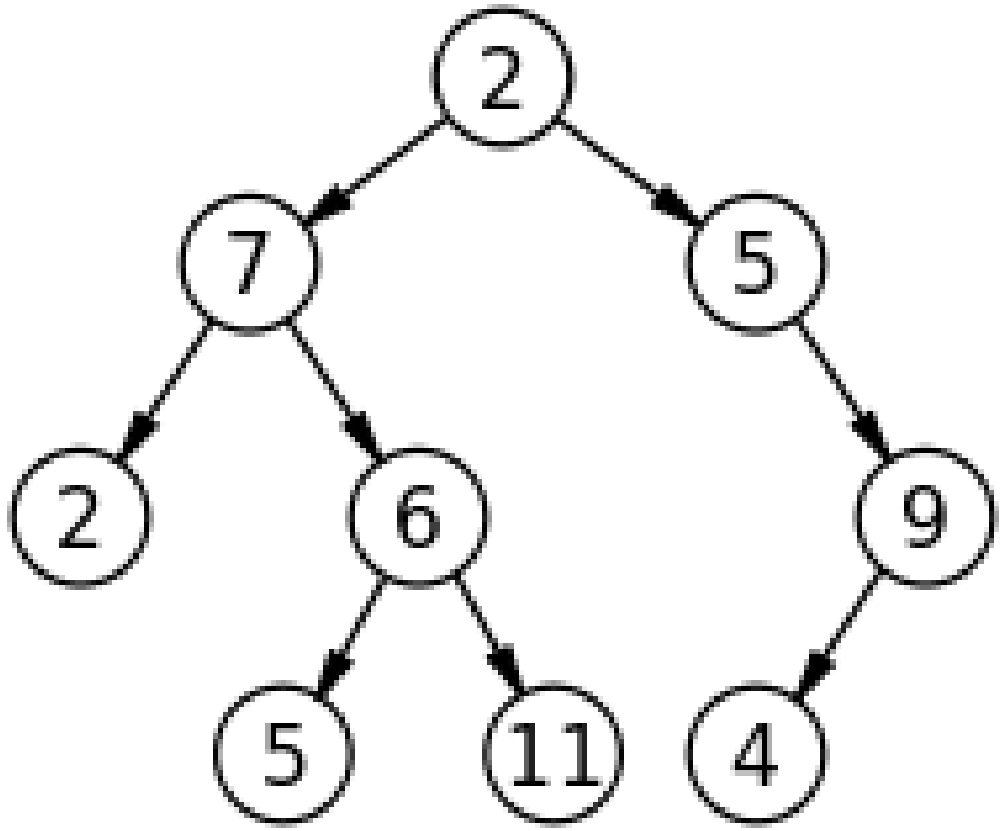
```
"""
A TreeList is either None or a Python list with 3 elements, where
  --- element 0 is a value
  --- element 1 is a TreeList
  --- element 2 is a TreeList
"""

def inorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in order

    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
    >>> inorder(T)
    [4, 5, 2, 3, 1]
    """
```

post-order traversal

Visit post-order left subtree, then post-order right subtree, then root



exercise: code for postorder traversal

```
"""
A TreeList is either None or a Python list with 3 elements, where
  --- element 0 is a value
  --- element 1 is a TreeList
  --- element 2 is a TreeList
"""

def postorder(tl: 'TreeList') -> list:
    """
    Return list of values in tl in postorder

    >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
    >>> postorder(T)
    [4, 2, 1, 3, 5]
    """
```

general tree implementation

Python **list** class has way more methods and attributes than needed. Let's specialize on **Tree** ADT.

```
class Tree:
    def __init__(self: 'Tree',
                 value: object =None, children: list =None):
        """Create a node with value and any number of children"""

        self.value = value
        if not children:
            self.children = []
        else:
            self.children = children[:] # quick-n-dirty copy of list

    def __contains__(self: 'Tree' , value: object) -> bool:
        """True if Tree has a node with value
        """
        return (self.value == value or
                any([t.__contains__(value) for t in self.children]))
```

t = 0

add a string representation

NB repr(t) — t: tree.

Tree
- value
- list of children (children) → Trees.

Tree(value, [...])

```
def __repr__(self: 'Tree') -> str:
    """Return representation of Tree as a string"""
```

return 'Tree({}, {})'.format(repr(self.value), repr(self.children))

sum up the number of nodes

module-level function

Tree class
value — object
children — list of Trees

```
def count(t: Tree) -> int:
    """How many nodes in this Tree?

    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> count(tn1)
    9
    """
```

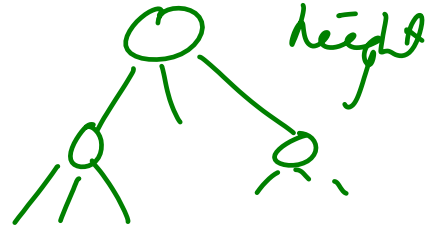
t = ?

vs

return 1 + sum([count(c) for c in t.children])

height of this tree?

height 0



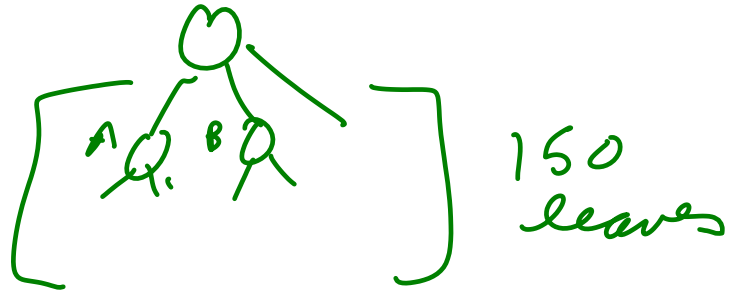

```
def height(t: Tree) -> int:
    """Return length of longest path of t
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> height(tn1)
    2
    """
    # 1 more edge than the maximum height of a child, except
    # what happens if there are no children?
```

[] in boolean context → False.

== []

return (0 if t.children is None else 1 + max([height(c) for c in t.children]))

how many leaves?




```
def leaf_count(t: Tree) -> int:
    """Return number of leaves in t

    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> leaf_count(tn1)
    6
    """
```

{you are welcome to write if — return — else return

return (sum([leaf_count(c) for c in t.children] if t.children else 1)

arity, or branching factor

```
def arity(t: Tree) -> int:
    """Maximum branching factor of tree T

    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
```