

recursive structures week 6

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Outline



recursion, natural and otherwise





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terminology

- set of nodes (possibly with values or labels), with directed edges between some pairs of nodes
- One node is distinguished as root
 Each non-root node has exactly one parent.
- A path is a sequence of nodes n_1, n_2, \ldots, n_k , where there is an edge from n_i to n_{i+1} . The length of a path is the number of edges in it
- There is a unique path from the root to each node. In the case of the root itself this is just n_1 , if the root is node n_1 .

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▶ There are no cycles — no paths that form loops.

more terminology

- ▶ leaf: node with no children
- ▶ internal node: node with one or more children
- subtree: tree formed by any tree node together with its descendants and the edges leading to them.
- height: Maximum path length in a tree. A node also defines a height, which is the maximum path length of the tree rooted at that node
- > arity, branching factor: maximum number of children for any node.

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pre-order traversal

Visit root, then pre-order left subtree, then pre-order right subtree



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exercise: code for preorder traversal

..... A TreeList is either None or a Python list with 3 elements, where --- element 0 is a value --- element 1 is a TreeList --- element 2 is a TreeList def preorder(tl: 'TreeList') -> list: Return list of values in tl in preorder Nonell >>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]] >>> preorder(T) preorder (teliz [5, 4, 3, 2, 1] ret

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in-order traversal

Visit in-order left subtree, then root, then in-order right subtree



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exercise: code for inorder traversal

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post-order traversal

Visit post-order left subtree, then post-order right subtree, then root



exercise: code for postorder traversal

```
"""
A TreeList is either None or a Python list with 3 elements, where
--- element 0 is a value
--- element 1 is a TreeList
--- element 2 is a TreeList
"""
def postorder(tl: 'TreeList') -> list:
"""
Return list of values in tl in postorder
>>> T = [5, [4, None, None], [3, [2, None, None], [1, None, None]]]
>>> postorder(T)
[4, 2, 1, 3, 5]
"""
```



general tree implementation

Python list class has way more methods and attributes than needed. Let's specialize on Tree ADT.

```
class Tree:
    def __init__(self: 'Tree',
                 value: object =None, children: list =None):
        """Create a node with value and any number of children"""
        self.value = value
        if not children:
            self.children = []
        else
            self.children = children[:] # quick-n-dirty copy of list
    def __contains__(self: 'Tree' , value: object) -> bool:
        """True if Tree has a node with value
        .....
        return (self.value == value or
                any([t.__contains__(value) for t in self.children]))
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