

- A2 - office hours - today 11-noon → behind BA1180
- today 6-8 pm BA2230 → BA4270
- tomorrow 11-1pm BA2230 (Help Centre)
- tomorrow 6-8 p.m. BA2230

CSC148 fall 2013

sorting big-oh
week 9

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BA4270 (behind elevators)

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Outline

more big-oh

running time analysis

Worst-case
running time

size of
problem

algorithm's behaviour over large input (size n) is common way to compare performance

constant: $c \in \mathbb{R}^+$ (some positive number)

logarithmic: $O(\log n)$

linear: cn (probably not the same c)

quadratic: cn^2

cubic: cn^3

exponential: $c2^n$

horrible: cn^n or $cn!$



case: $\lg n$

$$2^k = n \xrightarrow{h} \text{means } \lg n = k$$

this is the number of times you can divide n in half before reaching 1.

Definition.

▶ refresher: $a^b = c$ means $\log_a c = b$.

- ▶ this runtime behaviour often occurs when we “divide and conquer” a problem (e.g. binary search)
- ▶ we usually assume $\lg n$ (log base 2), but the difference is only a constant:

$$2^{\log_2 n} = n = 10^{\log_{10} n} \implies \log_2 n = \log_2 10 \times \log_{10} n$$

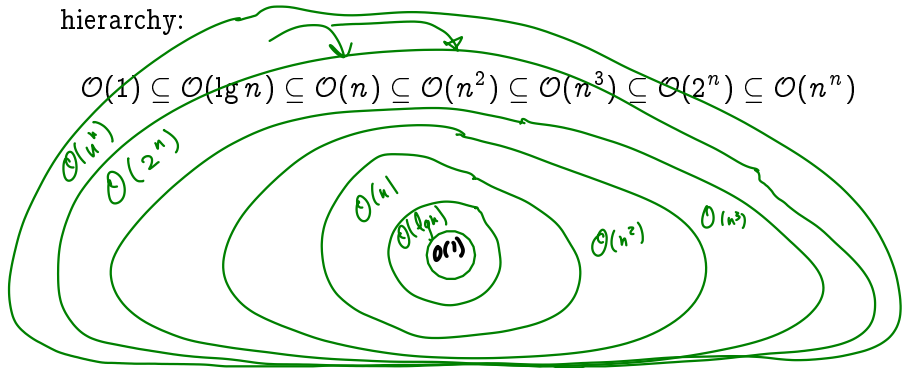
Handwritten notes: $2^{\log_2 n} = n$ and $10^{\log_{10} n} = n$ are circled in blue. Arrows point from the circled n to the boxed equations. Below the main equation, $2^{\log_2 n} = n$ and $10^{\log_{10} n} = n$ are written in green.

- ▶ so we just say $\mathcal{O}(\lg n)$.

hierarchy

Since big-oh is an **upper-bound** the various classes fit into a hierarchy:

$$\mathcal{O}(1) \subseteq \mathcal{O}(\lg n) \subseteq \mathcal{O}(n) \subseteq \mathcal{O}(n^2) \subseteq \mathcal{O}(n^3) \subseteq \mathcal{O}(2^n) \subseteq \mathcal{O}(n^n)$$



selection sort (review?)

performance

C → number of "steps" in for loop
X

$$n + (n-1) + (n-2) + \dots + 1$$

idea: for each position in the list, select the minimum item from that position on

$$\begin{array}{ccccccc} \downarrow & 1 & + & 2 & + & \dots & + & n \\ & n & + & n-1 & + & \dots & + & 2 & + & 1 \\ \downarrow & & & & & & & & & \downarrow \end{array}$$

$$\frac{(1+n) + (1+n) + \dots + (1+n)}{2} = \frac{n(n+1)}{2}$$

$$= \frac{1}{2}n^2 + \frac{n}{2}$$



