

CSC104 fall 2012

Why and how of computing week 7

Danny Heap

heap@cs.toronto.edu

BA4270 (behind elevators)

<http://www.cdf.toronto.edu/~heap/104/F12/>

416-978-5899

Text: **Picturing Programs**

Outline

convert to binary

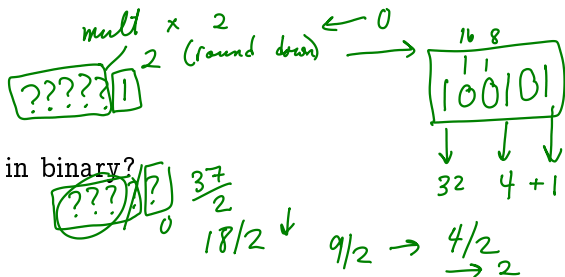
More numbers

characters, images, sound

Really crude encryption

Notes

number to binary



How do you write 37 in binary?

?????

- ▶ Suppose you knew it had six binary digits (bits), ??????. Does the fact that 37 is odd help you know whether the bit on the right is a 0 or 1?

- ▶ Suppose you know what the digit on the right is. What connection is there between the remaining bits, ?????, and $37/2$ (rounded down)?

Multiplication

multiply, shift, add

$$\begin{array}{r} 15 \\ \times 19 \\ \hline 135 \\ 135 \\ \hline 285 \end{array}$$

285

$$\begin{array}{r} 256 \\ 29 \\ \hline 285 \end{array}$$

X

$$\begin{array}{r} 10001101 \\ \times 10001101 \\ \hline 256 \quad 16 \quad 8+4+1 \end{array}$$

Once we can add non-negative integers, we can multiply them with a small number of additional operations. Consider the binary multiplication table:

| | | |
|---|---|---|
| × | 0 | 1 |
| 0 | 0 | 0 |
| 1 | 0 | 1 |

$$\begin{array}{r} 1111 \\ \times 10011 \\ \hline 11111 \\ 111110 \\ 1000000 \\ 10000000 \\ \hline 100001001 \end{array}$$

I use this to multiply the binary representations of 15 and 19. Other arithmetic functions are implemented as particular circuits.

Negative numbers, fractions

reassign some bits

32 bits,
use 1 digit for
+/-

Sometimes the left-most bit is used to represent + (as 0) or - (as 1). Another (efficient) scheme is called **two's complement**

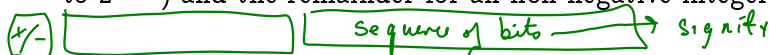
1 _____ negative #
0 _____ positive #

$$3.2 = \frac{32}{10}$$

In base 10, by shifting a number right (past the decimal point) we multiply it by $1/10$. In binary, we shift right past the binary point, and reduce by $1/2$.

$$1.01 = \frac{5}{4} = \leftarrow \frac{1}{4}$$
$$101 = 5$$
$$10.1 = 5/2 = 2\frac{1}{2}$$

A common scheme, called **IEEE floating point**, uses 64 bits (binary digits): one for the sign, 11 for the magnitude (from 2^{-1022} to 2^{1023}) and the remainder for an non-negative integer.



Enough numbers!

what about text?

iterate.

7 bits is enough to represent 128 values: upper- and lower-case latin characters, 10 numerals, some punctuation, and special control characters.

| Bits | | b ₆ | b ₅ | b ₄ | b ₃ | b ₂ | b ₁ | b ₀ | 0 0 0 0 1 0 1 1 0 1 1 1 1 1 | | | | | | | |
|------|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------------------|-----|----|---|---|---|---|-----|
| | | b ₆ | b ₅ | b ₄ | b ₃ | b ₂ | b ₁ | b ₀ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | NUL | DLE | SP | 0 | @ | P | · | p |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | SOH | DC1 | ! | 1 | A | Q | a | q |
| 0 | 0 | 1 | 0 | 2 | STX | DC2 | " | 2 | B | R | b | r | | | | |
| 0 | 0 | 1 | 1 | 3 | ETX | DC3 | # | 3 | C | S | c | s | | | | |
| 0 | 1 | 0 | 0 | 4 | EOT | DC4 | \$ | 4 | D | T | d | t | | | | |
| 0 | 1 | 0 | 1 | 5 | ENQ | NAK | % | 5 | E | U | e | u | | | | |
| 0 | 1 | 1 | 0 | 6 | ACK | SYN | & | 6 | F | V | f | v | | | | |
| 0 | 1 | 1 | 1 | 7 | BEL | ETB | ' | 7 | G | W | g | w | | | | |
| 1 | 0 | 0 | 0 | 8 | BS | CAN | (| 8 | H | X | h | x | | | | |
| 1 | 0 | 0 | 1 | 9 | HT | EM |) | 9 | I | Y | i | y | | | | |
| 1 | 0 | 1 | 0 | 10 | LF | SUB | * | | J | Z | j | z | | | | |
| 1 | 0 | 1 | 1 | 11 | VT | ESC | + | | K | [| k | [| | | | |
| 1 | 1 | 0 | 0 | 12 | FF | FC | . | < | L | \ | l | l | | | | |
| 1 | 1 | 0 | 1 | 13 | CR | GS | - | = | M |] | m |] | | | | |
| 1 | 1 | 1 | 0 | 14 | SO | RS | . | > | N | ^ | n | ^ | | | | |
| 1 | 1 | 1 | 1 | 15 | SI | US | / | ? | O | _ | o | _ | | | | DEL |

More than 110,000 characters can be specified with **unicode** (using more than 7 bits each).

What about images

how do those pixels glow so?

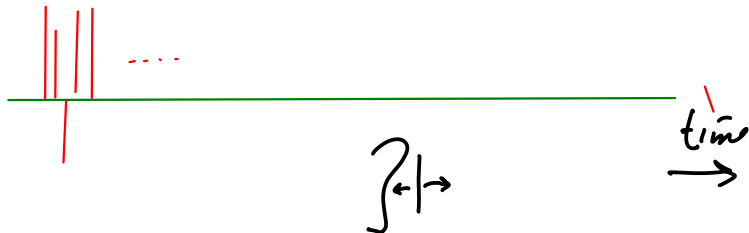
Computers represent images as rectangular arrays of glowing pixels. There are various schemes to determine what colour each pixels glows, one is **rgba**, which is what we use in picturing-programs

$$256 \times 256 \times 256 \times 256$$

Handwritten diagram showing the expansion of the expression above. The first '256' has a downward arrow pointing to 'r'. The second '256' has a downward arrow pointing to 'g'. The third '256' has a downward arrow pointing to 'b'. The fourth '256' has a downward arrow pointing to 'a'. A green line is drawn above the expression, connecting the top of the first '256' to the top of the fourth '256'.

Each colour is a value between 0 and 255 (inclusive). This allows 2^{32} over 4 billion colours. The “alpha” band represents opacity from clear (0) to opaque (255).

Sound

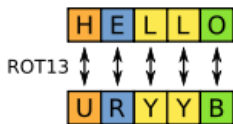
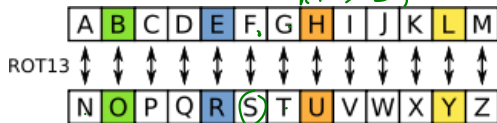


All the complexity of a room full of instruments can be simulated using a stream of numbers. After all, you can model sound as the displacement of your eardrum one way or the other. That's what the **WAV sound format**, a variety of **LPCM** (Linear Pulse Code Modulation) does.

a blast from the past

really bad text encryption

$A \rightarrow M$, add 13
 $N \rightarrow z$, subtract
 $a \rightarrow m$, add 13
 $n \rightarrow z$, subtract.



Encrypt "S"TRING" \rightarrow "FGEVAT"

rot13 as an algorithm

- ▶ What is given, what's required?

unencrypted string
encrypted (rot13) version

- ▶ Redo the last step for a single character

→ unencrypted character
→ encrypted character

- ▶ What is a really simple rule (or set of rules) for (rot13 c), where c is some character?
- ▶ It might help to know that characters #\A through #\Z have ascii encodings 65 through 90.

more rot13

- ▶ What about characters that aren't in `#\A` through `#\Z`?
- ▶ What about lower-case characters?
- ▶ How do we get from characters to strings of characters?

reversing strings

Give step-by-step instructions to reverse “string”

- ▶ given/required?
- ▶ check-expect some small examples?
- ▶ try to write down a recipe

How do you recognize a palindrome, such as “rotor” or “ACTAGATCA”?

- ▶ given/required?
- ▶ check-expect a small example or two
- ▶ try to state the recipe

Notes