QUESTION 1. [17 MARKS]

PART (A) [5 MARKS]

Given an example of an algorithm that you use outside of CSC104. Explain the algorithm in sufficient detail that somebody who had never performed it could do so.

SOLUTION: I make tea in my kitchen, as follows:

1. Find the loose tea in the cupboard on the west side of the kitchen, otherwise return to step 1 tomorrow.
2. Find a tablespoon in the drawer on the west side of the kitchen, otherwise return to step 1 tomorrow.
3. Find the cup in the cupboard on the north side of the kitchen, otherwise return to step 1 tomorrow.
4. Find the kettle on the stove on the west side of the kitchen, otherwise return to step 1 tomorrow.
5. Fill the kettle at the tap on the north side of kitchen with roughly 250 mL of water.
6. Put the kettle on the stove, turn the burner knob counter-clockwise as far as it will go. If the burner doesn't light, turn the knob clockwise as far as it will go and return to step 1 tomorrow.
7. When the kettle boils, turn the knob clockwise as far as it will go, then put a tablespoon of tea and boiling water into cup (being careful not to overflow).

PART (B) [4 MARKS]

What tools (mental or physical) are needed to carry out the grade-school algorithm for multiplication (done in lecture) and understand the result?

SOLUTION:

1. Positional number system.
2. Some way of recording results (pencil, chalk, pen).
3. Times table from 0×0 through 9×9.
4. Addition algorithm.

PART (C) [4 MARKS]

Convert the base-ten numbers below to their (unsigned) binary equivalents, and then perform the arithmetic operation indicated. Show your work when you carry out addition or multiplication in binary.

i) $47 + 53$

SOLUTION: 47 is 101111 in binary, and 53 is 110101 in binary, so

$$\begin{array}{r} 101111 \\ +110101 \\ \hline 1100100 \end{array}$$

ii) 5×9

SOLUTION: 5 is 101 in binary, and 9 is 1001, so

$$\begin{array}{r} 1001 \\ \times 101 \\ \hline 1001 \\ 0000 \\ +1001 \\ \hline 101101 \end{array}$$

PART (D) [4 MARKS]

Fill in the missing column of the following logical operation, where 0 is interpreted as false and 1 is interpreted as true. No explanation is required.

b_1	b_2	NOT (b_1 OR b_2)
0	0	
0	1	
1	0	
1	1	

SOLUTION:

b_1	b_2	NOT (b_1 OR b_2)
0	0	1
0	1	0
1	0	0
1	1	0

QUESTION 2. [10 MARKS]

PART (A) [4 MARKS]

Examine the fragment of a gnumeric spreadsheet below. Notice that the formula in cell A2 is $= A1 * \$B\$1$. Suppose you copy A2 to A3 and A4 (using the mouse to drag the lower-right corner down). What will appear in cells A2, A3, and A4? Explain why.

SOLUTION: A2 will contain 4, since the formula says to multiply the contents of A1 times B1. A3 will contain 8, since when the formula is copied A1 increments to A2, while the $ ensures that B1 remains B1, so we multiply 4 times 2. A4 will contain 16, since the formula is incremented to A3 times B1, or 8 times 2.

PART (B) [4 MARKS]

Suppose the file *words* contains only the following four entries:

```
warthog
answer
giraffe
xylophone
```

Which words are produced by the following commands. Explain why.

i) `grep -w [wagx].*[m-z] words`

SOLUTION: answer, since this is the only word that begins with one of the characters in the set [wagx], followed by zero or more characters, and ending with a letter in the set [m-z].

ii) `grep -w [gw].....[ge] words`

SOLUTION: warthog and giraffe, since these are the only words in the file that begin with a character in the set [gw], followed by five characters, followed by a character in the set [ge].

PART (C) [2 MARKS]

Recall the algorithm from lecture and assignment 1 to find the GCD of two non-negative integers. Why do we say this is an efficient algorithm?

SOLUTION: The algorithm takes very few steps (empirically less than twenty on the numbers we tried), and can be described very briefly.

QUESTION 3. [5 MARKS]

Consider the figure drawn (in bold lines) on a grid below. How many line segments begin and end on grid crossings? Explain your answer as fully as possible.

SOLUTION: Each of the horizontal lines has 7 line segments of length 1 (since there are 7 possible starting points), 6 line segments of length 2 (there are 6 possible starting points), 5 of length 3 (5 possible starting points), 4 of length 4 (4 possible starting points), 3 of length 5 (3 possible starting points), 2 of length 6 (2 possible starting points), and 1 of length 7 (1 possible starting point). The sum $7+6+5+4+3+2+1$ is $(7\times 8)/2$ (done in class). There are 8 horizontal lines and 8 vertical lines in the figure, and they all have the same number of line segments, so the total is:

$$2 \times 8 \times (8 \times 7)/2 = 64 \times 7 = 448.$$

Total Marks = 32