CSC104 fall 2012

Why and how of computing

week 4

Danny Heap
heap@cs.toronto.edu
BA4270 (behind elevators)
http://www.cdf.toronto.edu/~heap/104/F12/
416-978-5899

Text: Picturing Programs

Outline

Multiplication

multiply, shift, add

mult by 2, shift left

in

store a small mult table

11

110 ←

Once we can add non-negative integers, we can multiply them with a small number of additional operations. Consider the binary multiplication table:

×	0	1
0	0	0
1	0	1

1111

10011

x

1111

11110

00000

0000

1111

I use this to multipy the binary representations of 15 and 19.

Other arithmetic functions are implemented as particular circuits.

1 + 4 + 8 + 16 + 256

29

285

1000 111 01

See Notes (last slide)

Negative numbers, fractions

reassign some bits

Sometimes the left-most bit is used to represent + (as 0) or − (as 1). Another (efficient) scheme is called two's complement

In base 10, by shifting a number right (past the decimal point) we multiply it by 1/10. In binary, we shift right past the binary point, and reduce by 1/2.

A common scheme, called IEEE floating point, uses 64 bits (**b**inary digits): one for the sign, 11 for the magnitude (from 2^{-1022} to 2^{1023}) and the remainder for an non-negative integer.

Enough numbers!

what about text?

2^7

7 bits is enough to represent 128 values: upper- and lower-case latin characters, 10 numerals, some punctuation, and special control characters.

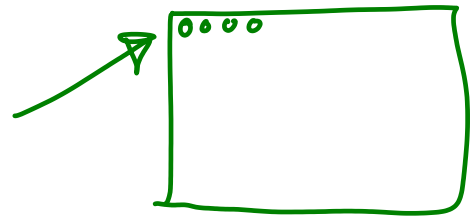
				b_7	0	0	0	0	1	1	1	1
				b_6	0	0	1	1	0	0	1	1
				b_5	0	1	0	1	0	1	0	1
Bits b_4	b_3	b_2	b_1	Row ↓ / Column →	0	1	2	3	4	5	6	7
0	0	0	0	0	NUL	DLE	SP	0	@	P	`	p
0	0	0	1	1	SOH	DC1	!	1	A	Q	a	q
0	0	1	0	2	STX	DC2	"	2	B	R	b	r
0	0	1	1	3	ETX	DC3	#	3	C	S	c	s
0	1	0	0	4	EOT	DC4	$	4	D	T	d	t
0	1	0	1	5	ENQ	NAK	%	5	E	U	e	u
0	1	1	0	6	ACK	SYN	&	6	F	V	f	v
0	1	1	1	7	BEL	ETB	'	7	G	W	g	w
1	0	0	0	8	BS	CAN	(	8	H	X	h	x
1	0	0	1	9	HT	EM	)	9	I	Y	i	y
1	0	1	0	10	LF	SUB	*	:	J	Z	j	z
1	0	1	1	11	VT	ESC	+	;	K	[	k	{
1	1	0	0	12	FF	FC	,	<	L	\	l	|
1	1	0	1	13	CR	GS	-	=	M	]	m	}
1	1	1	0	14	SO	RS	.	>	N	^	n	~
1	1	1	1	15	SI	US	/	?	O	_	o	DEL

8 multiples of bits

More than 110,000 characters can be specified with unicode (using more than 7 bits each).

What about images

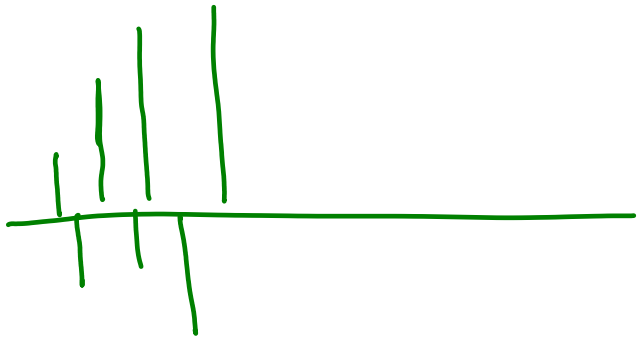
how do those pixels glow so?

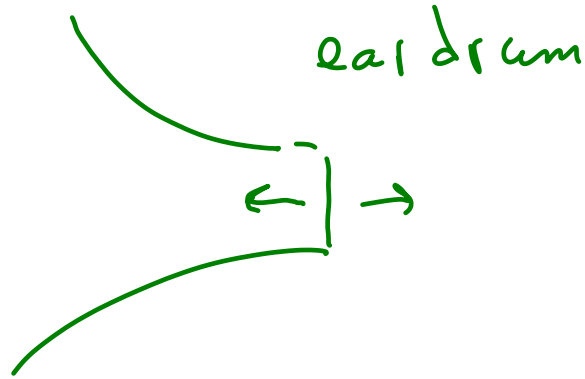


Computers represent images as rectangular arrays of glowing pixels. There are various schemes to determine what colour each pixels glows, one is rgba, which is what we use in `picturing-programs`

Each colour is a value between 0 and 255 (inclusive). This allows 2^{32}, over 4 billion colours. The "alpha" band represents opacity from clear (0) to opaque (255).

Sound





All the complexity of a room full of instruments can be simulated using a stream of numbers. After all, you can model sound as the displacement of your eardrum one way or the other. That's what the WAV sound format, a variety of LPCM (Linear Pulse Code Modulation) does.

44 100/sec (CD)

Notes

5 - 3 in 4-bit 2s complement. The left-most bit is reserved for sign (+/-) in this case (0 - +, 1 - -).

5 is written 0101 → sign bit

-3 is produced by taking +3: 0011
then "toggle" bits 1100
then add 1 + 1

11 01 ← -3??!

Now, use the full adder
+5 - 3 →
= +2

```
 1  1
    0101
 +  1101
 -------
    0010   +2!
```

(last carry thrown out).