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complete induction

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Using *Introduction to the Theory of Computation*,
Section 1.3
Outline

Principle of complete induction

Examples of complete induction
Complete Induction
another flavour needed

Every natural number greater than 1 has a prime factorization

Try some examples

How does the factorization of 8 help with the factorization of 9?
More dominoes

\[(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)\]

If all the previous cases always implies the current case then all cases are true
Every natural number greater than 1 has a prime factorization

\((\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)\)
Every natural number greater than 1 has a prime factorization

\((\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)\)
A tree is a directed graph.

A non-empty tree has a root node, \( r \), such that there is exactly one path from \( r \) to any other node.

If a tree has an edge \((u, v)\), then \( u \) is \( v \)'s parent, \( v \) is \( u \)'s child.

Two nodes with the same parent are called siblings.

A node with no children is called a leaf. A non-leaf is called an internal node.

Binary trees have nodes with \( \leq 2 \) children, and children are labelled either left or right.

Internal nodes of full binary trees have 2 children.
Tree examples

know your trees...
Every full binary tree, except the zero tree, has an odd number of nodes

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Every full binary tree, except the zero tree, has an odd number of nodes

\((\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)\)
Every rectangular array of chocolate $m \times n$ squares can be broken up with "breaks"

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$(\forall n \in \mathbb{N}, \langle P(0), \ldots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$
After a certain natural number $n$, every postage can be made up by combining 3— and 5— cent stamps.