CSC236 fall 2012
regular languages, regular expressions

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Using Introduction to the Theory of Computation,
Chapter 7
regular expressions, regular languages

notes
they’re equivalent:

\[ L = L(M) \text{ for some DFSA } M \iff L = L(M') \text{ for some NFSA } M' \iff L = L(R) \text{ for some regular expression } R \]

step 1: convert \( L(M) \) to \( L(R) \), eliminate states
they’re equivalent:

\[ L = L(M) \] for some DFSA \( M \) \( \iff \) \( L = L(M') \) for some NFSA \( M' \) \( \iff \)

\[ L = L(R) \] for some regular expression \( R \)

step 1: convert \( L(M) \) to \( L(R) \), eliminate states
equivalence...

state elimination recipe for state $q$

1. $s_1 \ldots s_m$ are states with transitions to $q$, with labels $S_1 \ldots S_m$
2. $t_1 \ldots t_n$ are states with transitions from $q$, with labels $T_1 \ldots T_n$
3. $Q$ is any self-loop on $q$
4. Eliminate $q$, and add (union) transition label $S_iQ^*T_j$ from $s_i$ to $t_j$. 

\[ R^* S (Q + TR^*S)^* \]
equivalence:
step 2: convert $L(R)$ to $L(M)$:
start with $\emptyset$, $\varepsilon$, $a \in \Sigma$

\[
\begin{align*}
S &\rightarrow O & M\emptyset \\
S &\rightarrow O & M\varepsilon \\
S &\rightarrow O & M_a, \quad a \in \Sigma \\
S &\rightarrow O \quad \alpha \rightarrow O
\end{align*}
\]
equivalence... 

step 2.5: convert $L(R)$ to $L(M)$: union, concatenation, stars

Assume we have $M_T$, $M_S$, accept $L(T)$ and $L(S)$.

$M(T+S)$  

$s \rightarrow \varepsilon$  

M_T \rightarrow \varepsilon \rightarrow M_S

$M(TS)$

$M(T^*)$

$s \rightarrow \varepsilon$  

$(0+1)^*0(0+1)(0+1)(0+1)$

$(0+1)(000)^*$
\[ L = \{ \epsilon, 10, 1100, 111000, \ldots \} \]

\[ L = \{ \epsilon, 10, 1010, 101010, \ldots \} \]

**Proof (contradiction)**

That no DFA accepts \( L \).

Suppose \( M \) accepts \( L \).

Then \( M \) has finite # of states, \( k \in \mathbb{N}^+ \).

What happens with \( 1^k \epsilon^{k+1} = S \ 1^k \epsilon^{k+1} \)?

in process \( S \), \( M \) proceeds

\[ q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow \ldots \rightarrow q_{k+1} \rightarrow q_{k+2} \rightarrow \ldots \rightarrow q_{2k+2} \]

Some \( q_j \) visited twice.

\[ S \rightarrow q_j \rightarrow q_{k+2} \rightarrow \ldots \rightarrow q_{2k+2} \]
\[ l = \{ 2, 3, 5, 7, 11, 13 \} \quad | \quad |151| \text{ is prime} \]

Colin
Norman
Feyyaz