This test consists of 3 questions on 6 pages (including this one). When you receive the signal to start, please make sure that your copy of the test is complete.

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write “I cannot answer this question,” on.

Good Luck!
QUESTION 1. [22 marks]
Consider the following python functions and definitions, where \( L \) is a list and \( P \) is a boolean function. Note: feel free to ask about how python works, since you are being tested on logic, not programming.

\[
\text{def q1}(L, P) : \text{return False not in } [P(x) \text{ for } x \text{ in } L]
\]

\[
\text{def q2}(L, P) : \text{return False in } [P(x) \text{ for } x \text{ in } L]
\]

\[
\text{def q3}(L, P) : \text{return True not in } [P(x) \text{ for } x \text{ in } L]
\]

\[
\text{def q4}(L, P) : \text{return True in } [P(x) \text{ for } x \text{ in } L]
\]

\( L1 = [0, 1, 2, 3, 4, 5] \)

\[
\text{def P1}(x) : \text{return } x < 2
\]

\[
\text{def P2}(x) : \text{return } x < 3
\]

\[
\text{def P3}(x) : \text{return (not P2(x)) or P1(x)}
\]

PART (A) [6 marks]
Write the name of each function q1--q4 beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. \( \forall x \in L, P(x) \) \quad q1

2. \( \exists x \in L, P(x) \) \quad q4

3. \( \forall x \in L, \neg P(x) \) \quad q3

4. \( \exists x \in L, \neg P(x) \) \quad q2

mutual negations

mutual negations
PART (B) [16 marks]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

\[ p_1 : x < 2 \]
\[ p_3 : x < 3 \Rightarrow x < 2 \]

1. q1(L1, P1) False, \( p_1(3) \) is False, so \( \forall x \in L_1, p_1(x) \) is False
2. q2(L1, P1) True, \( p_1(3) \) is False, so \( \exists x \in L_1, \neg p_1(x) \) is True
3. q3(L1, P1) False, \( p_1(1) \) is True, so \( \forall x \in L_1, \neg p_1(x) \) is False
4. q4(L1, P1) True - this is the negation of \( \neg q_3 \)
5. q1(L1, P3) False. \( 2 < 3 \land \neg (2 < 2) \), so \( \forall x \in L_1, p_3(x) \Rightarrow p_2(x) \) is False
6. q2(L1, P3) True - this is the negation of \( q_1 \)
7. q3(L1, P3) False, \( 4 < 3 \Rightarrow 4 < 2 \) is true (vacuous), so \( \forall x \in L_1, \neg (x < 3 \Rightarrow x < 2) \) is False
8. q4(L1, P3) True - negation of \( \neg q_3 \)
QUESTION 2.  [10 marks]

PART (A)  [5 marks]

Consider the following symbolic statement:

\[ S_1 : \forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon \]

1. Write the negation of the symbolic statement \( S_1 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( x > \delta \) or \( x^3 > \varepsilon \).

\[ \exists \varepsilon \in \mathbb{R}^+, \forall \delta \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \land \neg (x^3 > \varepsilon) \]

2. Which is true, statement \( S_1 \) or its negation? Briefly explain your reasoning.

\( S_1 \) is true. Given \( \varepsilon > 0 \), pick \( \delta = \frac{3\sqrt[3]{\varepsilon}}{2} \). Then

\[ \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > (\frac{3\sqrt[3]{\varepsilon}}{2})^3 = \varepsilon \]

\( \neg S_1 \) is false.

PART (B)  [5 marks]

Now consider the symbolic statement:

\[ S_2 : \exists \delta \in \mathbb{R}^+, \forall \varepsilon \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon \]

1. Write the negation of the symbolic statement \( S_2 \), in such a way that the negation symbol \( \neg \) applies only to predicates such as \( x > \delta \) or \( x^3 > \varepsilon \).

\[ \forall \delta \in \mathbb{R}^+, \exists \varepsilon \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \land \neg (x^3 > \varepsilon) \]

2. Which is true, statement \( S_2 \) or its negation? Briefly explain your reasoning.

\( S_2 \) is true. Given \( \delta > 0 \), pick \( \varepsilon = 8\delta^3 \) and \( \chi = 2\delta \). Then \( \chi = 2\delta > \delta \land \chi^3 = 8\delta^3 \leq \varepsilon = 8\delta^3 \).
Question 3. [6 marks]

Suppose $F$ is the set of functions, $D(f)$ means "$f$ is differentiable," and $C(f)$ means "$f$ is continuous." Consider the following statement:

$S3$: "Every function is not differentiable unless it is continuous."

Write the contrapositive, converse, and the negation of $S3$ symbolically. Note: in this course we translate "unless" as "if not".

**Contrapositive**

$$\forall f \in F, \neg D(f) \Rightarrow \neg C(f)$$

**Converse**

$$\forall f \in F, \neg D(f) \Rightarrow \neg C(f)$$

**Negation**

$$\exists f \in F, \neg C(f) \land D(f).$$
This page is left (nearly) blank to accommodate work that wouldn’t fit elsewhere.

# 1: _____/22
# 2: _____/10
# 3: _____/ 6

TOTAL: _____/38