1. Describe an appropriate reduction to show that the following function is not computable, where $P$ is any program that takes exactly one input $x$. Don't forget to argue that your reduction is correct!

$$\text{allstop}(P) = \begin{cases} 
\text{True} & \text{if } P(x) \text{ halts for every input } x, \\
\text{False} & \text{otherwise}.
\end{cases}$$

For a contradiction, assume that $\text{allstop}(P)$ is computable.

Consider the following program.

```python
def h(P, x):
    def P1(y):
        return P(x)
    return allstop(P1)
```

Then for all programs $P$ and inputs $x$:

- $h(P, x)$ returns True if $\text{allstop}(P1)$ returns True
  - if $P1(y)$ halts for every input $y$
  - if $P(x)$ halts;

- $h(P, x)$ returns False if $\text{allstop}(P1)$ returns False
  - if $P1(y)$ does not halt for some input $y$
  - if $P(x)$ does not halt.

(This works because $P1(y)$ halts for all inputs $y$ iff $P(x)$ halts.)

In other words, $h(P, x)$ computes function $\text{halt}$, a contradiction!

Hence, by contradiction, $\text{allstop}$ is not computable.
2. Describe an appropriate reduction to show that the following function is not computable, where $P$ is any program that takes exactly one input $x$. Don't forget to argue that your reduction is correct!

$$\text{steps}(P,x) = \begin{cases} 
\text{the number of lines of code executed by } P \text{ on input } x, & \text{if } P(x) \text{ halts,} \\
0 & \text{otherwise.}
\end{cases}$$

For a contradiction, assume that $\text{steps}(P,x)$ is computable.

Consider the following program.

```python
def h(P, x):
    return \text{steps}(P, x) > 0
```

Then for all programs $P$ and inputs $x$:

- $h(P, x)$ returns True if $\text{steps}(P, x)$ returns a positive integer if $P(x)$ halts;
- $h(P, x)$ returns False if $\text{steps}(P, x)$ returns 0 if $P(x)$ does not halt.

In other words, $h(P, x)$ computes function $\text{halt}$, a contradiction!

Hence, by contradiction, $\text{steps}$ is not computable.