CSC165 winter 2013
Mathematical expression

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Course notes, chapter 4
Prove $3n^2 + 2n \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{ f : \mathbb{N} \to \mathbb{R}_{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2 \}$
Special case? what happens if you add a constant?

Prove $3n^2 + 2n + 5 \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{ f : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2 \}$
Look at the leading term

Prove: $7n^6 - 5n^4 + 2n^3 \in \mathcal{O}(6n^8 - 4n^5 + n^2)$

Use $\mathcal{O}(6n^8 - 4n^5 + n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^\geq | \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq c(6n^8 - 4n^5 + n^2)\}$
how to prove $n^3 \not\in O(3n^2)$?

Negate $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow n^3 \leq c3n^2$}
non-polynomials

Big-oh statements about polynomials are pretty easy to prove: $f \in \mathcal{O}(g)$ exactly when the highest-degree term of $g$ is no smaller than the highest-degree term of $f$.

What about functions such as $\log(n)$ or $3^n$? Logarithmic functions are in big-Oh of any polynomial, whereas exponential functions (with a base bigger than 1) are not in big-Oh of any polynomial. How do you prove such things?
Prove \(2^n \not\in O(n^2)\)

Use \(\lim_{n \to \infty} \frac{2^n}{n^2}\)

Do you know anything about the ratio \(\frac{2^n}{n^2}\), as \(n\) gets very large? How do you evaluate:

\[
\lim_{n \to \infty} \frac{2^n}{n^2}
\]

If the limit evaluates to \(\infty\), then that’s the same as saying:

\[
\forall c \in \mathbb{R}^+, \exists n' \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n' \Rightarrow \frac{2^n}{n^2} > c
\]

Once your enemy hands you a \(c\), you can choose an \(n'\) with the required property.