QUESTION 1.  [8 marks]

Suppose $S$ is the set of students in your section of CSC165, $\mathbb{R}$ is the set of real numbers, and the predicate $M(x, y)$ means that the mass of $x$ is $y$ grams. Consider two symbolic statements:

$s1: \forall x \in S, \exists y \in \mathbb{R}, M(x, y)$.

$s2: \exists y \in \mathbb{R}, \forall x \in S, M(x, y)$.

PART (A)  [2 marks]

Write the symbolic expression for the negation of $s1$, and the symbolic expression for the negation of $s2$, in such a way that the negation symbol appears to the right of all quantifiers.

Solution:

$\neg s1 : \exists x \in S, \forall y \in \mathbb{R}, \neg M(x, y)$

$\neg s2 : \forall y \in \mathbb{R}, \exists x \in S, \neg M(x, y)$

PART (B)  [4 marks]

Which of $s1$ or $s2$ are true, which are false? Explain your answers.

Solution: $s1$ is true, since it claims that every student has a mass. $s2$ is false, since it claims that there is a single mass that every student has.

PART (C)  [2 marks]

Do your answers to PART B change if $S$ is the set containing only one element (me)? Explain why or why not.

Solution: $s1$ remains the same (true) since every element of the set has a mass. $s2$ is true for a singleton set, since there is a single mass that every element of the set has: its mass.

QUESTION 2.  [8 marks]

Consider the four methods m1–m4 below, and the boolean method P which hasn’t been implemented yet.

```java
static boolean m1(Object[] A){
    for (int i = 0; i < A.length; ++i) { if (P(A[i])) { return true; }}
    return false;
}

static boolean m2(Object[] A) {
    for (int i = 0; i < A.length; ++i) { if (P(A[i])) { return false; }}
    return true;
}

static boolean m3(Object[] A) {
    for (int i = 0; i < A.length; ++i) { if (!P(A[i])) { return true; }}
    return false;
}
```

Student #: .................................  Page 1 of 3 contaminants
static boolean m4(Object[] A) {
    for (int i = 0; i < A.length; ++i) {
        if (!P(A[i])) {
            return false;
        }
    }
    return true;
}

PART (A)  [4 marks]

Beside each comment below, write the name of the method, or methods, that best implement(s) that comment.

1. For every Object o in A, P(o) is true. Solution: m4

2. For some Object o in A, P(o) is true. Solution: m1

3. For every Object o in A, P(o) is false. Solution: m2

4. For some Object o in A, P(o) is false. Solution: m3

PART (B)  [4 marks]

Which comments (if any) are negations of other comments? Explain your answer.

Solution: Comment 4 is the negation of comment 1, since (moving negation in), \( \neg(\forall o \in A, P(o)) \) is \( \exists o \in A, \neg P(o) \). Comment 3 is the negation of comment 2, since (moving negation in) \( \neg(\exists o \in A, P(o)) \) is \( \forall o \in A, \neg P(o) \).

QUESTION 3.  [10 marks]

PART (A)  [6 marks]

Re-write each of the following symbolic expressions as equivalent expressions using only parentheses, A, B, C, D, \lor, \land, and \lnot.

1. \( A \Rightarrow (B \Rightarrow (C \Rightarrow D)) \)
   Solution:
   \( \neg A \lor (\neg B \lor (\neg C \lor D)) \)

2. \( ((A \Rightarrow B) \Rightarrow C) \Rightarrow D \)
   Solution:
   \( ((\neg A \lor B) \land \neg C) \lor D \)

3. \( (A \Rightarrow B) \Rightarrow (C \Rightarrow D) \)
   Solution:
   \( (A \land \neg B) \lor (\neg C \lor D) \)
PART (B)  [4 marks]
Use your answer from PART A to show that the following are equivalent to $A \Rightarrow (B \Rightarrow (C \Rightarrow D))$.

1. $B \Rightarrow (C \Rightarrow (A \Rightarrow D))$
   
   Solution: $A \Rightarrow (B \Rightarrow (C \Rightarrow D))$ is equivalent to (from PART A) $-A \lor (-B \lor (-C \lor D))$ is equivalent to (by associativity and commutativity of $\lor$) $-B \lor (-C \lor (-A \lor D))$ equivalent to $B \Rightarrow (C \Rightarrow (A \Rightarrow D))$.

2. $B \Rightarrow (A \Rightarrow (C \Rightarrow D))$

   Solution: $A \Rightarrow (B \Rightarrow (C \Rightarrow D))$ is equivalent to (from PART A) $-A \lor (-B \lor (-C \lor D))$ is equivalent to (by associativity and commutativity of $\lor$) $-B \lor (-A \lor (-C \lor D))$ is equivalent to $B \Rightarrow (A \Rightarrow (C \Rightarrow D))$.

QUESTION 4.  [6 marks]

Let $U$ be some universe containing sets $P$, $Q$, $R$, and let $P(u)$ mean $u \in P$, $Q(u)$ means $u \in Q$, and $R(u)$ mean $u \in R$. For each statement in precise symbolic notation, shade the corresponding Venn diagram to indicate which regions can be non-empty without making the statement false. You earn more marks the more regions you shade correctly, and marks will be deducted if you shade regions that make the corresponding statement false. No justification required.

1. $\forall u \in U, -(P(u) \Rightarrow Q(u)) \lor -(Q(u) \Rightarrow R(u))$.
   
   Solution: Shade regions $(P - Q) \cup (Q - R)$

2. $\forall u \in U, (P(u) \Rightarrow Q(u)) \land (Q(u) \Rightarrow R(u)) \land (R(u) \Rightarrow P(u))$.
   
   Solution: Shade regions $PQ \cap R$ and the complement of $P \cup Q \cup R$.

Total Marks = 32