CSC 165

proof

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Chains with $\wedge$ or $\lor$

Chains of antecedents consequents break up in asymmetrical ways. Use truth tables, venn diagrams, or rules for manipulating predicates to show

$$((P \Rightarrow R_1) \wedge (P \Rightarrow R_2)) \iff (P \Rightarrow (R_1 \wedge R_2))$$

Notice that things switch when the conjunction is at the other end of the implication

$$((R_1 \Rightarrow Q) \wedge (R_2 \Rightarrow Q)) \iff ((R_1 \lor R_2) \Rightarrow Q)$$
odd example

The square of an odd number is odd. Prove:

$$\forall n \in \mathbb{N}, n \text{ odd } \Rightarrow n^2 \text{ odd}.$$
scratch
Claim:
Prove that for every pair of non-negative real numbers \((x, y)\), if \(x\) is greater than \(y\), then the geometric mean, \(\sqrt{xy}\), is less than the arithmetic mean, \((x + y)/2\).

Assume \(x, y \in \mathbb{R}^{\geq 0}\) # general elements of \(\mathbb{R}^{\geq 0}\)
Assume \(x > y\) # assume antecedent.
Then \(x - y > 0\) # subtract \(y\) from both sides.
\(x^2 + y^2 - 2xy > 0\) # square
\(x^2 + y^2 + 2xy > 4xy\) # add \(4xy\) to both sides
\((x + y)^2 > 4xy\) # factor
\((x + y) > 2\sqrt{xy}\) # take non-neg \(\sqrt{\cdot}\) since \(x, y\) non-neg.
\(x + y > 2\sqrt{xy}\) # divide by 2 — consequent.
\(\frac{x + y}{2} > \sqrt{xy}\) # divide by 2 — consequent

Then \(x > y \Rightarrow \sqrt{xy} < \frac{x + y}{2}\) # consequent assumed

Then \(\forall x, y \in \mathbb{R}^{\geq 0}, x > y \Rightarrow \sqrt{xy} < \frac{x + y}{2}\) # intro A.
\[ \sqrt{xy} \leq \frac{x+y}{2} \]

\[ 2\sqrt{xy} \leq x+y \]

\[ 4\ xy \leq x^2 + y^2 + 2\ xy \]

\[ 0 \leq x^2 + y^2 - 2\ xy \]

\[ 0 \leq (x-y)^2 \]

compare

\[ \implies \]

\[ \implies \]

\[ \implies \]

\[ \implies \]
Prove that for any natural number \( n \), \( n^2 \) odd implies that \( n \) is odd.

Assume \( n \in \mathbb{N} \)

Assume \( n^2 \) odd \# antecedent

Then \( \exists k \in \mathbb{N}, n^2 = 2k + 1 \) \# definition of odd

Pick \( k \in \mathbb{N} \), \( n^2 = 2k + 1 \) \# since it exist

Assume \( n \) is even \# antecedent of contrapositive

Almost same as several slide ago.

Then \( n^2 \) is even \# \( \exists j \in \mathbb{N}, n = 2j \)

Then \( n^2 \text{ odd} \Rightarrow n \text{ odd} \) \# introduced implication

Conclude \( \forall n \in \mathbb{N}, n^2 \text{ odd} \Rightarrow n \text{ odd} \) \# introduced \( \forall \).
scratch