CSC165, Winter 2010  
Exercise 1

due: January 19th, 10pm

These exercises are meant to resemble material we’ve covered in lecture on quantification and implication. Each question is worth the same amount, so please complete whatever you can. You will receive 20% of the relevant marks for any portion you leave blank, or write “I cannot answer this” on.

You must write up your exercise alone. You may discuss general approaches to solutions with other students, but be very careful not to take notes (electronic, or on paper), and to wait an hour after such a conversation before writing up your solution.

Submit your work as a PDF file e1.pdf to the CDF secure web site:

https://www.cdf.toronto.edu/students/

1. Consider the four python functions below. In each case L is expected to be a list of numbers, and P is expected to be a boolean function taking one numerical argument.

   def qp1(L, P) : return False in [P(x) for x in L]
   def qp2(L, P) : return True in [P(x) for x in L]
   def qp3(L, P) : return False not in [P(x) for x in L]
   def qp4(L, P) : return True not in [P(x) for x in L]

   With the following definitions of L and P, [qp1(L,P), qp2(L,P), qp3(L,P), qp4(L,P)] evaluates to a particular four-element list of values from {True, False}:

   def P(x) : return x > 5
   L = []
   [qp1(L,P), qp2(L,P), qp3(L,P), qp4(L,P)]
   [False, False, True, True]

   There are 15 more possible four-element lists of values from {True, False}. For each four-element list, either provide an L and a P that produce it when [qp1(L,P), qp2(L,P), qp3(L,P), qp4(L,P)] is evaluated, or else explain why it is impossible to find a suitable L and P.

2. Each of the sentences below expresses an implication of the form “If P, then Q.” For each case (i) identify the antecedent P and the consequent Q, and (ii) express a counterexample to the implication (even if such a counterexample doesn’t exist).

   (a) Unless you’re here for an argument, you’ll be disappointed.
   (b) Beggars can’t be choosers.
   (c) Don’t stay in the kitchen if you can’t stand the heat.
(d) You're part of the problem unless you're part of the solution.
(e) You're not part of the solution unless you're not part of the problem.
(f) A function is not continuous, or it is integrable.
(g) A house divided against itself cannot stand.

3. \( \mathbb{N} \) is the set of natural numbers \( \{0, 1, 2, 3, \ldots\} \) (in Computer Science we include 0). Consider the four open sentences below:

\[
\forall n \in \mathbb{N}, P(n) \Rightarrow Q(n) \\
\exists n \in \mathbb{N}, P(n) \Rightarrow Q(n) \\
\forall n \in \mathbb{N}, Q(n) \Rightarrow P(n) \\
\exists n \in \mathbb{N}, Q(n) \Rightarrow P(n)
\]

Evaluate each of the four sentences above using each of the four definitions of predicates \( P \) and \( Q \) below. Briefly explain your evaluation of each of the sixteen cases.

(a) \( P(n) : n < 0 \) \( Q(n) : n^2 < 0 \)
(b) \( P(n) : n < 0 \) \( Q(n) : n^2 > 17 \)
(c) \( P(n) : n > 3 \) \( Q(n) : n^2 < 0 \)
(d) \( P(n) : n > 3 \) \( Q(n) : n^2 > 17 \)