Use the proof structure from this course to prove or disprove. You may assume that all functions map natural numbers to non-negative real numbers.

1. \((f \in \Omega(g) \land f \in \Omega(g)) \Rightarrow f \in \Omega(g + h)\)

2. \((f \in \Omega(g) \land f \in \Omega(h)) \Rightarrow f \in \Omega(g \cdot h)\)

3. \((f \in O(g) \land g \in O(h)) \Rightarrow f \in O(h)\)

4. \((f \in O(g) \land f \in O(h)) \Rightarrow f \in O(\min(g, h))\)
SOLUTIONS

These are not fully written out, but contain the gist of each solution.

1. The claim is true. If asymptotically \( f(n) \geq c_1 g(n) \) and \( f(n) \geq c_2 h(n) \), then set \( c = (c_1 + c_2)/2 \), or something similar.

2. The claim is false. A counterexample is \( f(n) = g(n) = h(n) = n \).

3. The claim is false. A counterexample is \( f(n) = 1, g(n) = n^2, h(n) = n \).

4. The claim is true. If asymptotically \( f(n) \leq c_1 g(n) \) and \( f(n) \leq c_2 h(n) \), then set \( c = \max(c_1, c_2) \). Notice that \( \max(c_1, c_2) \times \min(g(n), h(n)) \geq \min(c_1 g(n), c_2 h(n)) \geq f(n) \)