QUESTION 1. [10 marks]
Recall these python functions from lecture:

\[
\begin{align*}
\text{def quant1(L1, L2) : return False in } [x \text{ in L2 for } x \text{ in L1}] \\
\text{def quant2(L1, L2) : return True in } [x \text{ in L2 for } x \text{ in L1}] \\
\text{def quant3(L1, L2) : return False not in } [x \text{ in L2 for } x \text{ in L1}] \\
\text{def quant4(L1, L2) : return True not in } [x \text{ in L2 for } x \text{ in L1}]
\end{align*}
\]

PART (A) [4 marks]
Write the name of each function above beside the comment(s) that best describes the condition for which the function returns True.

1. \( \forall x \in L1, x \in L2 \) Solution: quant3
2. \( \exists x \in L1, x \in L2 \) Solution: quant2
3. \( \forall x \in L1, x \notin L2 \) Solution: quant4
4. \( \exists x \in L1, x \notin L2 \) Solution: quant1

PART (B) [6 marks]
For each output (i)–(vi), either devise lists \( L1 \) and \( L2 \) so that the python expression

\[
[\text{quant1}(L1,L2), \text{quant2}(L1,L2), \text{quant3}(L1,L2), \text{quant4}(L1,L2)]
\]

evaluates to that output, or else explain why it is impossible to devise such lists.

(i) \([T,T,F,F]\) Solution: \( L1 = [0,1], L2 = [0] \)

(ii) \([T,F,T,F]\) Solution: Not possible, since \( q1(L1,L2) = \neg q3(L1,L2) \)

(iii) \([T,F,F,T]\) Solution: \( L1 = [0], L2 = [] \)

(iv) \([F,T,T,F]\) Solution: \( L1 = [0], L2 = [0] \)

(v) \([F,T,F,T]\) Solution: Not possible, since \( q1(L1,L2) = \neg q3(L1,L2) \)

(vi) \([F,F,T,T]\) Solution: \( L1 = [], L2 = [] \)
**Question 2.** [12 marks]

**Part (A) [8 marks]**

Consider the claim:

\[ \forall x \in U, (P(x) \lor Q(x)) \Rightarrow R(x) \]

...where \( P(x) \) means \( x \in P \), \( Q(x) \) means \( x \in Q \), and \( R(x) \) means \( x \in R \). Write an F in the regions of the Venn diagram below that would provide a counterexample to the claim if they were occupied. You earn one mark for each correctly placed F, and one mark for each correctly omitted F.

![Venn Diagram](image)

Solution: Write an F in each region of \( P \cup Q \) that is outside \( R \). These three Fs correspond to counterexamples that satisfy \( (P(x) \lor Q(x)) \land \neg R(x) \)

**Part (B) [4 marks]**

Devise elements of set \( U \) and meaning of predicates \( P(x) \) and \( Q(x) \) where \( S1 \) is false but \( S2 \) is true. Then devise elements of set \( U \) and meaning of predicates \( P(x) \) and \( Q(x) \) where \( S1 \) is true but \( S2 \) is false:

\[
S1: \quad \forall x \in U, P(x) \Rightarrow Q(x) \\
S2: \quad \exists x \in U, P(x) \Rightarrow Q(x)
\]

Solution: Example # 1: \( U = \{0,1\} \quad P(x) : x > 0 \quad Q(x) : x > 1 \)

Example # 2: \( U = 0 \quad P(x) : x > 0 \quad Q(x) : x > 1 \)
QUESTION 3.  [10 marks]

Define set $H$ as the set of humans, and predicates $P(h)$ as "$h$ is part of the problem," $S(h)$ as "$h$ is part of the solution." For each English statement below, rewrite the statement and its negation in symbolic form, where the $\neg$ symbol, if it is used, is as close as possible to the predicates as possible. For each symbolic statement below, rewrite the statement and its negation in English.

PART (A)  [2 marks]

Some humans are part of the solution provided they are not part of the problem.

Solution: rewrite: $\exists h \in H, \neg P(h) \Rightarrow S(h)$, negation: $\forall h \in H, \neg P(h) \land \neg S(h)$

PART (B)  [2 marks]

$\forall h \in H, \neg P(h) \land S(h)$.

Solution: rewrite: Every human is not part of the problem and is part of the solution. negation: Some human is part of the problem if they are part of the solution.

PART (C)  [2 marks]

Each human is not part of the solution unless they are not part of the problem.

Solution: rewrite: $\forall h \in H, P(h) \Rightarrow \neg S(h)$ negation: $\exists h \in H, P(h) \land S(h)$

PART (D)  [2 marks]

$\exists h_1 \in H, \exists h_2 \in H, P(h_1) \land S(h_2)$.

Solution: rewrite: There's somebody who is part of the problem, and there's somebody who is part of the solution. negation: If anybody is part of the problem, then nobody is part of the solution.

PART (E)  [2 marks]

Any human being is part of the solution only if they are not being part of the problem.

Solution: rewrite: $\forall h \in H, S(h) \Rightarrow \neg P(h)$ negation: $\exists h \in H, S(h) \land P(h)$
QUESTION 4. [6 marks]

Define $C$ as the set of courses at U of T, $S$ as the set of students at U of T, and $E(s,c)$ as student $s$ is enrolled in course $c$. For each pair of statements below, explain either why they are equivalent, or give an example that shows they are different.

**Part (a) [2 marks]**

$(S1) \ \ \ \ \forall c \in C, \exists s \in S, E(s,c)$

$(S2) \ \ \ \ \forall s \in S, \exists c \in C, E(s,c)$

Solution: They aren't equivalent. If all students were enrolled in some course or another, but one course had nobody enrolled, then $(S1)$ would be false, whereas $(S2)$ would be true.

**Part (b) [2 marks]**

$(S3) \ \ \ \ \exists s \in S, \forall c \in C, E(s,c)$

$(S1) \ \ \ \ \forall c \in C, \exists s \in S, E(s,c)$

Solution: They aren't equivalent. If every course has at least one student enrolled, but no student is enrolled in all courses, then $(S1)$ is true whereas $(S3)$ is false.

**Part (c) [2 marks]**

$(S4) \ \ \ \ \exists s \in S, \exists c \in C, E(s,c)$

$(S5) \ \ \ \ \exists c \in C, \exists s \in S, E(s,c)$

Solution: They are equivalent — they both say that there is at least one student enrolled in at least one course.
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